New generalized sets in N-topological spaces

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Abstract. By introducing the topological generalized closed sets into N-topological space, this paper establishes that the union of τ_i-generalized closed sets need not be $N\tau$-generalized closed set. Further, the collection of $N\tau$-$\tilde{g}$ closed sets form a topology. Apart from this, $N\tau$-$\tilde{g}$ closed sets are characterized by means of $N\tau^{\#}gs$-kernel.

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1 Introduction

The concept of semi-open sets and generalized closed sets were initiated by Norman Levine [5,6] and also established their fundamental properties. O.Njastad [9] developed α-open sets and investigated its relationship with other open sets. Mashhour et al. [7,8] characterized pre-open sets and α-open sets with their continuous functions. Abd El-Monsef et al. [1] defined β-open sets with the properties of β-continuous mappings. J.Dontchev [2] evolved the concept of generalized semi-pre closed sets and derived their properties. P. Sundaram et al. [10] characterized the semi-generalized closed sets and their mappings. Lellis Thivagar et al. [3] discovered a geometrical structure of N-topological space with the N-topological open sets. Further Lellis Thivagar and Arockia Dasan [4] established some weak forms of open sets in N-topological space along with their mappings. In this paper, we discuss various kinds of generalized closed sets in N-topological spaces and establish their relationship. We also find that the $N\tau$-$\tilde{g}$ closed sets forms a topology which places between $N\tau$-closed and $N\tau$-generalized closed sets.

2 Preliminaries

In this section we recall some known definitions and results of N-topological space and weak open sets that will be used in the following sections. By the space $(X, N\tau)$, we mean, N-topological space with N-topology on X with no separation axioms are assumed unless specifically stated.

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Definition 2.1 (3)**.** Let X be a non empty set, $\tau_1,\ \tau_2,\ \ldots\ ,\ \tau_N$ be N-arbitrary topologies defined on X. Then the collection $N\tau = \{S \subseteq X : S = (\bigcup_{i=1}^{N} A_i) \cup (\bigcap_{i=1}^{N} B_i),\ A_i, B_i \in \tau_i\}$, is said to be N-topology if it satisfying the following axioms:

(i) $X, \emptyset \in N\tau$.

(ii) $\bigcup_{i=1}^{\infty} S_i \in N\tau$ for all $\{S_i\}_{i=1}^{\infty} \in N\tau$.

(iii) $\bigcap_{i=1}^{n} S_i \in N\tau$ for all $\{S_i\}_{i=1}^{n} \in N\tau$.

Then the ordered pair $(X, N\tau)$ is called an N-topological space on X and the elements of the collection $N\tau$ are known as $N\tau$-open sets on X. A subset A of X is said to be $N\tau$-closed on X if the complement of A is $N\tau$-open on X. The set of all $N\tau$-open sets on X and the set of all $N\tau$-closed sets on X are respectively denoted by $N\tau O(X)$ and $N\tau C(X)$.

Definition 2.2 (3)**.** Let $(X, N\tau)$ be an N-topological space and S be a subset of X. Then

(i) the $N\tau$-interior of S is defined as $N\tau\text{-}int(S) = \cup\{G : G \subseteq S$ and G is $N\tau$-open$\}$.

(ii) the $N\tau$-closure of S is defined as $N\tau\text{-}cl(S) = \cap\{F : S \subseteq F$ and F is $N\tau$-closed$\}$.

Definition 2.3 (4)**.** A subset A of N-topological space $(X, N\tau)$ is said to be

(i) $N\tau\alpha$-open if $A \subseteq N\tau\text{-}int(N\tau\text{-}cl(N\tau\text{-}int(A)))$.

(ii) $N\tau$ semi-open if $A \subseteq N\tau\text{-}cl(N\tau\text{-}int(A))$.

(iii) $N\tau$ pre-open if $A \subseteq N\tau\text{-}int(N\tau\text{-}cl(A))$.

(iv) $N\tau\beta$-open if $A \subseteq N\tau\text{-}cl(N\tau\text{-}int(N\tau\text{-}cl(A)))$.

The complement of above open sets are called respective closed sets. The family of $N\tau$-α (resp. $N\tau$-semi, $N\tau$-pre and $N\tau$-β) open sets is denoted by $N\tau\alpha O(X)$ (resp. $N\tau SO(X)$, $N\tau PO(X)$ and $N\tau\beta O(X)$).

Theorem 2.1 (4)**.** *In an N-topological space $(X, N\tau)$, the following are true:*

(i) every $N\tau$-open set is $N\tau$-α open.

(ii) every $N\tau$-α open set is both $N\tau$-semi and $N\tau$-pre open, vice versa.

(iii) every $N\tau$-semi open set is $N\tau$-β open.

(iv) every $N\tau$-pre open set is $N\tau$-β open.

3 Generalized closed sets in N-topological spaces

This section introduce the classical generalized closed sets into N-topological space. We also state that the union of τ_i-generalized closed sets need not be $N\tau$-generalized closed set and establish their relationships.

Definition 3.1. A subset A of N-topological space $(X, N\tau)$ is said to be

(i) $N\tau$ generalized-closed (briefly $N\tau g$-closed) if $N\tau$-$cl(A) \subseteq U$ whenever $A \subseteq U$and U is $N\tau$-open in $(X, N\tau)$.

(ii) $N\tau\alpha$ generalized-closed (briefly $N\tau\alpha g$-closed) if $N\tau$-$\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $N\tau$-open in $(X, N\tau)$.

(iii) $N\tau$ generalized α-closed (briefly $N\tau g\alpha$-closed) if $N\tau$-$\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $N\tau\alpha$-open in $(X, N\tau)$.

(iv) $N\tau$ generalized semi-closed (briefly $N\tau gs$-closed) if $N\tau$-$scl(A) \subseteq U$ whenever $A \subseteq U$ and U is $N\tau$-open in $(X, N\tau)$.

(v) $N\tau$ semi generalized-closed (briefly $N\tau sg$-closed) if $N\tau$-$scl(A) \subseteq U$ whenever $A \subseteq U$ and U is $N\tau$ semi-open in $(X, N\tau)$.

(vi) $N\tau\hat{g}$-closed if $N\tau$-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $N\tau$ semi-open in $(X, N\tau)$.

(vii) $N\tau^*g$-closed if $N\tau$-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $N\tau\hat{g}$-open in $(X, N\tau)$.

(viii) $N\tau^{\#}g$-semi closed (briefly $N\tau^{\#}gs$-closed) if $N\tau$-$scl(A) \subseteq U$ whenever $A \subseteq U$ and U is $N\tau^*g$-open in $(X, N\tau)$.

(ix) $N\tau\tilde{g}$-closed if $N\tau$-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $N\tau^{\#}gs$-open in $(X, N\tau)$.

The complement of above N-topological generalized closed set is called respective generalized open sets. The set of all $N\tau g$-closed (resp. $N\tau\alpha g$-closed, $N\tau g\alpha$-closed, $N\tau gs$-closed, $N\tau sg$-closed, $N\tau\hat{g}$-closed, $N\tau^*g$-closed, $N\tau^{\#}gs$-closed, $N\tau\tilde{g}$-closed) sets of $(X, N\tau)$ is denoted by $N\tau GC(X)$ (resp. $N\tau\alpha GC(X)$, $N\tau G\alpha C(X)$, $N\tau GSC(X)$, $N\tau SGC(X)$, $N\tau\hat{G}C(X)$, $N\tau^*GC(X)$, $N\tau^{\#}GSC(X)$, $N\tau\tilde{G}C(X)$).

The following example illustrates the uniqueness of this paper namely the union of generalized closed sets of topological spaces $(X, \tau_1), (X, \tau_2), ..., (X, \tau_N)$ need not be a generalized closed set of N-topological space.

Example 3.2. If $N = 2$, $X = \{a, b, c\}$, consider $\tau_1 = \{\varnothing, X, \{a\}\}$ and $\tau_2 = \{\varnothing, X, \{a, b\}\}$, then $2\tau O(X) = \{\varnothing, X, \{a\}, \{a, b\}\}$, $\tau_1 GC(X) = \tau_1\alpha GC(X) = \tau_1 GSC(X) = \{\varnothing, X, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$, $\tau_2 GC(X) = \tau_2\alpha GC(X) = \tau_2 GSC(X) = \{\varnothing, X, \{c\}, \{a, c\}, \{b, c\}\}$, $2\tau GC(X) = \{\varnothing, X, \{c\}, \{a, c\}, \{b, c\}\}$, ${\tau_1}^*GC(X) = \{\varnothing, X, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$, ${\tau_2}^*GC(X) = \{\varnothing, X, \{c\}, \{a, c\}, \{b, c\}\}$, $\tau_1\tilde{G}C(X) = \tau_1\hat{G}C(X) = \{\varnothing, X, \{b, c\}\}$, $\tau_2\tilde{G}C(X) = \tau_2\hat{G}C(X) = \{\varnothing, X, \{c\}, \{a, c\}, \{b, c\}\}$, $2\tau GC(X) = \{\varnothing, X, \{c\}, \{a, c\}, \{b, c\}\}$, $2\tau\alpha GC(X) = 2\tau GSC(X) = \{\varnothing, X, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$, $2\tau\tilde{G}C(X) = 2\tau\hat{G}C(X) = \{\varnothing, X, \{c\}, \{b, c\}\}$ and $2\tau^*GC(X) = \{\varnothing, X, \{c\}, \{a, c\}, \{b, c\}\}$. Hence we observe that $\tau_1\tilde{G}C(X) \cup \tau_2\tilde{G}C(X) \neq 2\tau\tilde{G}C(X)$, $\tau_1 GC(X) \cup \tau_2 GC(X) \neq 2\tau GC(X)$, $\tau_1\alpha GC(X) \cup \tau_2\alpha GC(X) \neq 2\tau\alpha GC(X)$ and ${\tau_1}^*GC(X) \cup {\tau_2}^*GC(X) \neq 2\tau^*GC(X)$.

Example 3.3. If $N = 2$, $X = \{a, b, c\}$, consider $\tau_1 = \{\varnothing, X, \{a\}, \{a, b\}\}$ and $\tau_2 = \{\varnothing, X, \{a, c\}\}$, then $2\tau O(X) = \{\varnothing, X, \{a\}, \{a, b\}, \{a, c\}\}$, $\tau_1{}^{\#}GSC(X) = \{\varnothing, X, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$, $\tau_2{}^{\#}GSC(X) = \{\varnothing, X, \{b\}, \{a, b\}, \{b, c\}\}$, $2\tau^{\#}GSC(X) = \{\varnothing, X, \{b\}, \{c\}, \{b, c\}\}$. Hence we find that $\tau_1{}^{\#}GSC(X) \cup \tau_2{}^{\#}GSC(X) \neq 2\tau^{\#}GSC(X)$.

By the following lemma we over come the above hurdles under certain conditions.

Lemma 3.1. *(i) If every $N\tau$-open set is τ_i-open, then every τ_i-g closed set is $N\tau$-g closed for $i = 1, 2, ..., N$.*

(ii) If every $N\tau$-open set is τ_i-open and $N\tau$-scl$(A) \subseteq \tau_i$-scl(A), then every τ_i-gs closed set is $N\tau$-gs closed for $i = 1, 2, ..., N$.

(iii) If $\tau_1 SO(X) = \tau_2 SO(X) = ... = N\tau SO(X)$, then every τ_i-sg closed set is $N\tau$-sg closed for $i = 1, 2, ..., N$.

(iv) If every $N\tau$-open set is τ_i-open and $N\tau$-$\alpha cl(A) \subseteq \tau_i$-$\alpha cl(A)$, then every τ_i-αg closed set is $N\tau$-αg closed for $i = 1, 2, ..., N$.

(v) If $\tau_1 \alpha O(X) = \tau_2 \alpha O(X) = ... = N\tau\alpha O(X)$, then every τ_i-$g\alpha$ closed set is $N\tau$-$g\alpha$ closed for $i = 1, 2, ..., N$.

(vi) If every $N\tau$-$\hat{g}$ open set is τ_i-$\hat{g}$ open, then every τ_i-$^*g$ closed set is $N\tau$-$*g$ closed.

(vii) If every $N\tau$-$*g$ open set is τ_i-$*g$ open and $N\tau$-scl$(A) \subseteq \tau_i$-scl(A), then every τ_i-$^{\#}gs$ closed set is $N\tau$-$^{\#}gs$ closed.

(viii) If every $N\tau$-$^{\#}gs$ open set is τ_i-$^{\#}gs$ open, then every τ_i-$\tilde{g}$ closed set is $N\tau$-$\tilde{g}$.

Proof. *Here we shall prove parts (i), (iii), (iv) and (viii). The remaining parts can be proved similarly.*

(i) Let A be a τ_i-g closed set and U be a $N\tau$-open set containing A, then by hypothesis, τ_i-cl$(A) \subseteq U$ implies $N\tau$-cl$(A) \subseteq U$. Therefore A is $N\tau$-g closed.

(iii) Let A be a τ_i-sg closed set and U be a $N\tau$-semi open set containing A, then τ_i-scl$(A) \subseteq U$ implies $N\tau$-scl$(A) \subseteq U$. Therefore A is $N\tau$-sg closed.

(iv) Let A be a τ_i-αg closed set and U be a $N\tau$-open set containing A, then τ_i-$\alpha cl(A) \subseteq U$ implies $N\tau$-$\alpha cl(A) \subseteq U$. Therefore A is $N\tau$-αg closed.

(viii) Let A be a τ_i-$\tilde{g}$ closed set and U be a $N\tau$-$^{\#}gs$-open set containing A, then τ_i-cl$(A) \subseteq U$ implies $N\tau$-cl$(A) \subseteq U$. Therefore A is $N\tau$-$\tilde{g}$ closed.

The following proposition is sibling of classical topological results.

Proposition 3.2. *Let $(X, N\tau)$ be an N-topological space, then every*

(i) $N\tau$-closed set is $N\tau$-g closed.

(ii) $N\tau$-semi closed set is $N\tau$-$^{\#}gs$ closed.

(iii) $N\tau$-α closed set is $N\tau$-$^{\#}gs$ closed.

(iv) $N\tau$-g *closed set is* $N\tau$-αg *closed.*

(v) $N\tau$-g *closed set is* $N\tau$-gs *closed.*

(vi) $N\tau$-sg *closed set is* $N\tau$-β *closed.*

(vii) $N\tau$-$g\alpha$ *closed set is* $N\tau$-*pre closed.*

The following examples illustrate that the converse of the above proposition need not be true.

Example 3.4. If $N = 6$, $X = \{a, b, c, d\}$, consider $\tau_1 = \{\varnothing, X, \{a\}\}$, $\tau_2 = \{\varnothing, X, \{c, d\}\}$, $\tau_3 = \{\varnothing, X, \{a, c, d\}\}$, $\tau_4 = \{\varnothing, X, \{b, c, d\}\}$, $\tau_5 = \{\varnothing, X, \{a\}, \{b, c, d\}\}$ and $\tau_6 = \{\varnothing, X, \{a\}, \{c, d\}, \{a, c, d\}\}$, then $6\tau O(X) = \{\varnothing, X, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$, $6\tau C(X) = \{\varnothing, X, \{a\}, \{b\}, \{a, b\}, \{b, c, d\}\}$ and $6\tau\tilde{G}C(X) = \{\varnothing, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$. Here the set $\{c\}$ is 6τ-pre closed and 6τ-β closed but not 6τ-sg closed and not 6τ-$g\alpha$ closed.

Example 3.5. If $N = 2$, $X = \{a, b, c\}$, consider $\tau_1 = \{\varnothing, X, \{a\}\}$ and $\tau_2 = \{\varnothing, X, \{a, b\}\}$, then $2\tau O(X) = \{\varnothing, X, \{a\}, \{a, b\}\}$. Here the set $\{b\}$ is 2τ-αg closed and 2τ-gs closed but not 2τ-g closed. Also the set $\{a, c\}$ is 2τ-g closed but not 2τ-closed.

Example 3.6. If $N = 2$, $X = \{a, b, c\}$, consider $\tau_1 = \{\varnothing, X, \{a\}\}$ and $\tau_2 = \{\varnothing, X, \{b, c\}\}$, then $2\tau O(X) = \{\varnothing, X, \{a\}, \{b, c\}\}$. Here the set $\{b\}$ is 2τ-$^{\#}gs$ closed but not 2τ-α closed and not 2τ-semi closed.

Proposition 3.3. *Let* $(X, N\tau)$ *be an* N*-topological space, then every*

(i) $N\tau$-*closed set is* $N\tau$-$\tilde{g}$ *closed.*

(ii) $N\tau$-$\tilde{g}$ *closed set is* $N\tau$-$\hat{g}$ *closed.*

(iii) $N\tau$-$\tilde{g}$ *closed set is* $N\tau$-g *closed.*

(iv) $N\tau$-$\tilde{g}$ *closed set is* $N\tau$-αg *closed.*

(v) $N\tau$-$\tilde{g}$ *closed set is* $N\tau$-sg *closed.*

(vi) $N\tau$-$\tilde{g}$ *closed set is* $N\tau$-β *closed.*

(vii) $N\tau$-$\tilde{g}$ *closed set is* $N\tau$-$g\alpha$ *closed.*

(viii) $N\tau$-$\tilde{g}$ *closed set is* $N\tau$-*pre closed.*

(ix) $N\tau$-$\tilde{g}$ *closed set is* $N\tau$-gs *closed.*

Proof. *Here we shall prove part* (i) *and* (iii) *only. The proof of the remaining parts are similar.*

(i) *If* A *is any* $N\tau$*-closed set in* $(X, N\tau)$ *and* U *is any* $N\tau$-$^{\#}gs$ *open set containing* A*. Then* $N\tau$-$cl(A) = A \subseteq U$ *implies* A *is a* $N\tau$-$\tilde{g}$ *closed.*

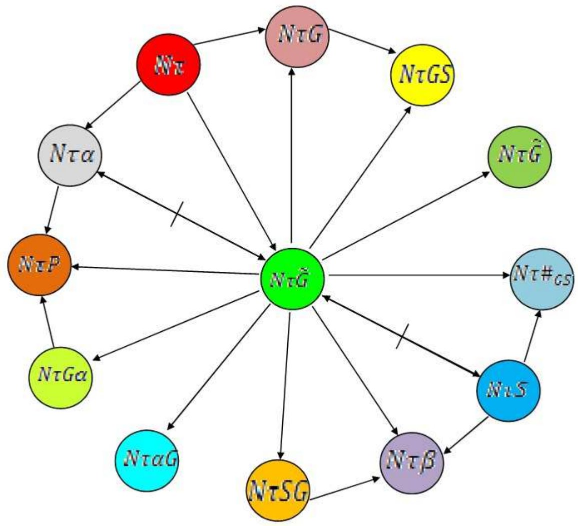

Figure 1: The relationship between N-topological generalized closed sets.

(iii) If A is any $N\tau$-$\tilde{g}$ closed set in $(X, N\tau)$ and U is any $N\tau$-open set containing A. Since every $N\tau$-closed set is $N\tau$-semi closed and every $N\tau$-semi closed set is $N\tau$-$^{\#}gs$ closed, then U is $N\tau$-$^{\#}gs$ open set containing A and so $N\tau$-$cl(A) \subseteq U$. Therefore A is a $N\tau$-g closed.

Example 3.7. The converse of the above proposition need not be true. For $N = 1$, $X = \{a, b, c\}$, $\tau = \tau_1 = \{\varnothing, X, \{a\}\}$. Then the set $\{b\}$ is $g\alpha$-closed, pre closed and gs-closed but not $\tilde{g}$-closed set. From example 3.4, the set $\{b, c\}$ is 6τ-$\tilde{g}$ closed but not 6τ-closed. Also from example 3.5, the set $\{b\}$ is 2τ-β closed and 2τ-sg closed but not 2τ-$\tilde{g}$ closed. By example 3.6 we know that the set $\{b\}$ is 2τ-g closed, 2τ-αg closed and 2τ-$\hat{g}$ closed but not 2τ-$\tilde{g}$ closed.

Remark 3.8. From the proposition 3.3, we observe that the set of all $N\tau$-$\tilde{g}$ closed sets placed between the set of all $N\tau$-closed and $N\tau$-g closed sets. It also placed between the set of all $N\tau$-closed and $N\tau$-$\hat{g}$ closed sets.

Remark 3.9. The following example states that the $N\tau$-$\tilde{g}$ closed set is independent of $N\tau$-α closed as well as $N\tau$-semi closed set.

Example 3.10. If $N = 2$, $X = \{a, b, c\}$, consider $\tau_1 = \{\varnothing, X, \{a, b\}\}$ and $\tau_2 = \{\varnothing, X\}$, then $2\tau O(X) = \{\varnothing, X, \{a, b\}\}$. Here the set $\{a, c\}$ is 2τ-$\tilde{g}$ closed but not 2τ-α closed and not 2τ-semi closed. From example 3.7, the set $\{b\}$ is α closed and semi closed but not $\tilde{g}$-closed.

Remark 3.11. The concept of generalized closed sets in N-topological space can be described in the diagram below where the reversible implication is not possible.

4 Characterization of $N\tau$-$\tilde{g}$ closed sets

In this section, we introduce $N\tau^{\#}gs$-$ker(A)$ and discuss the essential conditions for $N\tau$-$\tilde{g}$ closed sets in terms of $N\tau^{\#}gs$-$ker(A)$.

Definition 4.1. Let A be a subset of N-topological space $(X, N\tau)$, then $N\tau^{\#}gs$-$ker(A)$ is defined as the intersection of all $N\tau$-$^{\#}g$ semi open subsets of X containing A.

Lemma 4.1. *A subset A of an N-topological space $(X, N\tau)$ is $N\tau$-$\tilde{g}$ closed if and only if $N\tau$-$cl(A) \subseteq N\tau^{\#}gs$-$ker(A)$.*

Proof. Let A is $N\tau$-$\tilde{g}$ closed in $(X, N\tau)$, then for every $N\tau$-$^{\#}gs$ open set U containing A, $N\tau$-$cl(A) \subseteq U$. Assume $x \in N\tau$-$cl(A)$ and if $x \notin N\tau^{\#}gs$-$ker(A)$, then there exist a $N\tau$-$^{\#}gs$ open set U containing A such that $x \notin U$ implies $x \notin N\tau$-$cl(A)$. This is a contradiction to our assumption. Conversely, if $N\tau$-$cl(A) \subseteq N\tau^{\#}gs$-$ker(A)$ and U is a $N\tau$-$^{\#}gs$ open set containing A, then $N\tau$-$cl(A) \subseteq N\tau^{\#}gs$-$ker(A) \subseteq U$. Therefore A is $N\tau$-$\tilde{g}$ closed. $\square$ The following remarks are similar to the topological results of Dontchev [2].

Remark 4.2. Let x be a point of $(X, N\tau)$. Then $\{x\}$ is either $N\tau$-nowhere dense or $N\tau$-pre open.

Remark 4.3. In remark 4.2, the decomposition of an N-topological space $(X, N\tau)$, $X = X_1 \cup X_2$, where $X_1 = \{x \in X : \{x\}$ is $N\tau$-nowhere dense$\}$ and $X_2 = \{x \in X : \{x\}$ is $N\tau$-pre open$\}$.

Theorem 4.2. *For every subset A of an N-topological space $(X, N\tau)$, $X_2 \cap N\tau$-$cl(A) \subseteq N\tau^{\#}gs$-$ker(A)$.*

Proof. *Let $x \in X_2 \cap N\tau$-$cl(A)$, if $x \notin N\tau^{\#}gs$-$ker(A)$, then there exist a $N\tau$-$^{\#}gs$ open set U containing A such that $x \notin U$ implies $X \setminus U$ is a $N\tau$-$^{\#}gs$ closed set containing x. Since $x \in X_2 \cap N\tau$-$cl(A)$, $N\tau$-$int(N\tau$-$cl(\{x\})) \cap A \neq \emptyset$. Thus there is a point $y \in N\tau$-$int(N\tau$-$cl(\{x\})) \cap A \subseteq (X \setminus U) \cap A \subseteq (X \setminus U) \cap U$, which is a contradiction.*

Theorem 4.3. *A subset A of an N-topological space $(X, N\tau)$ is $N\tau$-$\tilde{g}$ closed if and only if $X_1 \cap N\tau$-$cl(A) \subseteq A$.*

Proof. *Let A be a $N\tau$-$\tilde{g}$ closed set, if $x \in X_1 \cap N\tau$-$cl(A)$ implies $x \in X_1$ and $x \in N\tau$-$cl(A)$. Since $x \in X_1$, $N\tau$-$int(N\tau$-$cl(\{x\})) = \emptyset$, $\{x\}$ is $N\tau$-semi closed. Since every $N\tau$-semi closed set is $N\tau$-$^{\#}gs$ closed, $\{x\}$ is $N\tau$-$^{\#}gs$ closed. If $x \notin A$ and $U = X \setminus \{x\}$, U is a $N\tau$-$^{\#}gs$ open set containing A and $N\tau$-$cl(A) \subseteq U$. This is a contradiction. Conversely, assume $X_1 \cap N\tau$-$cl(A) \subseteq A$ and $A \subseteq N\tau^{\#}gs$-$ker(A)$, then $X_1 \cap N\tau$-$cl(A) \subseteq N\tau^{\#}gs$-$ker(A)$. Now $N\tau$-$cl(A) = X \cap N\tau$-$cl(A) = (X_1 \cup X_2) \cap N\tau$-$cl(A) = (X_1 \cap N\tau$-$cl(A)) \cup (X_2 \cap N\tau$-$cl(A)) \subseteq N\tau^{\#}gs$-$ker(A)$. Thus by lemma 4.1, A is $N\tau$-$\tilde{g}$ closed.*

The proof of the following theorems are obvious from the above theorems.

Theorem 4.4. *The arbitrary intersection of $N\tau$-$\tilde{g}$ closed sets is $N\tau$-$\tilde{g}$ closed.*

Theorem 4.5. *If A is $N\tau$-$\tilde{g}$ closed set and B is $N\tau$-closed set, then $A \cap B$ is $N\tau$-$\tilde{g}$ closed.*

Theorem 4.6. *If A and B are two $N\tau$-$\tilde{g}$ closed sets, then $A \cup B$ is $N\tau$-$\tilde{g}$ closed set.*

Theorem 4.7. *If a set A is $N\tau$-$\tilde{g}$ closed, then $N\tau$-$cl(A) \setminus A$ contains no non empty $N\tau$-closed set.*

Proof. Suppose A is $N\tau$-$\tilde{g}$ closed in $(X, N\tau)$ and F be a $N\tau$-closed subset of $N\tau$-$cl(A) \setminus A$, then F^c is $N\tau$-${}^{\#}gs$ open and $A \subseteq F^c$. From the definition of $N\tau$-$\tilde{g}$ closed set it follows that $N\tau$-$cl(A) \subseteq F^c$ and $F \subseteq (N\tau\text{-}cl(A))^c$. Therefore $F \subseteq N\tau\text{-}cl(A) \cap (N\tau\text{-}cl(A))^c = \varnothing$ and so F is an empty set. □

Remark 4.4. The converse of the above theorem need not be true. By example 3.7, if $A = \{b\}$, then $cl(A) \setminus A = \{c\}$ does not contain non empty closed set also A is not $\tilde{g}$-closed in (X, τ).

Theorem 4.8. *A set A is $N\tau$-$\tilde{g}$ closed if and only if $N\tau$-$cl(A) \setminus A$ contains no non empty $N\tau$-${}^{\#}gs$ closed set.*

Proof. Assume A is $N\tau$-$\tilde{g}$ closed in $(X, N\tau)$ and F is a $N\tau$-${}^{\#}gs$ closed subset of $N\tau$-$cl(A) \setminus A$, then F^c is $N\tau$-${}^{\#}gs$ open containing A and $N\tau$-$cl(A) \subseteq F^c$. That is, $F \subseteq (N\tau\text{-}cl(A))^c$. Therefore $F \subseteq N\tau\text{-}cl(A) \cap (N\tau\text{-}cl(A))^c = \varnothing$ and F is an empty set. Conversely, let $N\tau$-$cl(A) \setminus A$ contains no non empty $N\tau$-${}^{\#}gs$ closed set and S be $N\tau$-${}^{\#}gs$ open set containing A in $(X, N\tau)$. Suppose $N\tau$-$cl(A)$ is not a subset of S, then $N\tau\text{-}cl(A) \cap S^c$ is a non empty $N\tau$-${}^{\#}gs$ closed subset of $N\tau$-$cl(A) \setminus A$, which is a contradiction. Therefore A is $N\tau$-$\tilde{g}$ closed in $(X, N\tau)$. □

Theorem 4.9. *If A is a $N\tau$-$\tilde{g}$ closed set and $A \subseteq B \subseteq N\tau\text{-}cl(A)$, then B is also $N\tau$-$\tilde{g}$ closed.*
Proof: *Let A be a $N\tau$-$\tilde{g}$ closed set and U be any $N\tau$-${}^{\#}gs$ open set containing B, then $N\tau\text{-}cl(B) \subseteq N\tau\text{-}cl(A) \subseteq U$. Therefore $N\tau\text{-}cl(B) \subseteq U$ and hence B is $N\tau$-$\tilde{g}$ closed.*

Theorem 4.10. *If A is $N\tau$-${}^{\#}gs$ open and $N\tau$-$\tilde{g}$ closed, then A is $N\tau$-closed.*

Proof. *Since A is $N\tau$-${}^{\#}gs$ open, $N\tau\text{-}cl(A) \subseteq A$. Therefore $N\tau\text{-}cl(A) = A$ and hence A is $N\tau$-closed.*

Theorem 4.11. *For each $x \in X$, either $\{x\}$ is $N\tau$-${}^{\#}gs$ closed, or $\{x\}^c$ is $N\tau$-$\tilde{g}$ closed.*
Proof: *If $\{x\}$ is not $N\tau$-${}^{\#}gs$ closed in $(X, N\tau)$, then $\{x\}^c$ is not $N\tau$-${}^{\#}gs$ open. The only $N\tau$-${}^{\#}gs$ open set containing $\{x\}^c$ is the set X itself. Therefore $N\tau\text{-}cl(\{x\}^c) \subseteq X$ and so $\{x\}^c$ is $N\tau$-$\tilde{g}$ closed in $(X, N\tau)$.*

5 Conclusion and future work

A N-topological space is a new space containing N-topologies defined on a non-empty set X. In this paper, we have attempted to define and establish N-topological generalized closed sets in N-topological space. Moreover, we have proved that the collection of $N\tau$-$\tilde{g}$ closed sets form a topology on X. This can be extended to real-life applications such as fuzzy topology, intuitionistic topology, nano topology, soft topology, etc. It also can open up doors to research areas like supra topology, digital topology, and so on.

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