Solving the nonsmooth bi-objective environmental and economic dispatch problem using smoothing functions

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Abstract. The Environmental and Economic Dispatch problem (EEDP) is a nonlinear Multi-objective Optimization Problem (MOP) which simultaneously satisfies multiple contradictory criteria, and it's a nonsmooth problem when valvepoint effects, multi-fuel effects and prohibited operating zones considered. It is an important optimization task in the fossil fuel fired power plant operation for allocating generation among the committed units such that fuel cost and pollution (emission level) are optimized simultaneously while satisfying all operational constraints. In this paper, we use smoothing functions with the gradient consistency property to approximate the nonsmooth MOP. Our approach is based on the smoothing method. In fact, we explain the convergence analysis of the smoothing method by using the approximate Karush-Kuhn-Tucker condition, which is necessary for a point to be a local weak efficient solution and it's also sufficient condition under convexity assumptions. Finally, we give an application of our approach for solving the bi-objective EEDP.

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1 Introduction

During the last decades the area of nonsmooth (nondifferentiable) Multiobjective Optimization Problems (MOP) has been extensively developed. The MOP refers to the process of simultaneously optimizing two or more real-valued objective functions. For nontrivial problems, no single point minimizes all given objective functions at once, so, the concept of optimality is to be replaced by the concept of Pareto optimality or efficiency. One should recall that a point is called Pareto optimal or efficient, if there is no different point with the same, or smaller, objective function values, such that there is a decrease in at least one objective function value. The nonsmooth MOP

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has applications in engineering [8], economics [12], mechanics [9] and other fields. For more details, see, for example, Miettinen [10].

In this paper, we concentrate on solving a class of nonsmooth MOP that include min, max, absolute value functions or composition of the plus function with smooth functions. The approximations are constructed based on the smoothing function for the plus function. For this end, we introduce the concept of approximate Karush-Kuhn-Tucker AKKT condition for the approximate multiobjective problem inspired by Giorrg. G. et al. [6] and we adapt it to prove the convergence analysis of the smoothing method, with feasible set defined by inequality constraints. Note that the AKKT condition has been widely used to define the stopping criteria of many practical contrained optimization algorithms [13, 2, 1]. The objective is to update the smoothing parameters to guarantee the convergence. We point out that Chen [3] has dealt with convergence analysis of the smoothing method (in the scalar case) by using a gradient method. At the end, we give an application of our approach in solving bi-objective Economic and Environmental Dispatching Problem (EEDP)[15, 5]. In fact, we transform the nonsmooth EEDP into a set of single-objective subproblems using the ϵ-constraint method. The objective function of the subproblems is smoothed and the subproblems are solved by the interior point barrier method.

This paper is organized as follows. In Section 2, we state the problem under consideration and we recall some useful basic notations. In Section 3, we define a class of smoothing composite functions by using the plus function. In Section 4, to explain the convergence analysis of the smoothing method, we use sequential AKKT. Finally, we show a numerical application in Section 5.

2 Basic notations and properties

The following notations are used throughout this paper. By $\langle \cdot, \cdot \rangle$, we denote the usual inner product on $\mathbb{R}^n$, and by $\| \cdot \|$ we denote its corresponding norm. Let $\mathbb{R}^p_+ = \{x \in \mathbb{R}^p \ : \ x_i \geq 0, \ i = 1, \cdots, p\}$, $\mathbb{R}^p_- = \{x \in \mathbb{R}^p \ : \ x_i \leq 0, \ i = 1, \cdots, p\}$, $\mathbb{R}^p_{++} = \{x \in \mathbb{R}^p \ : \ x_i > 0, \ i = 1, \cdots, p\}$ and $\mathbb{R}^p_{--} = \{x \in \mathbb{R}^p \ : \ x_i < 0, \ i = 1, \cdots, p\}$, we consider the partial orders $\succeq$ (respectively, $\preceq$) and $\succ$ (respectively, $\prec$), defined as $x \succeq y$ (respectively, $x \preceq y$) if and only if $x - y \in \mathbb{R}^p_+$ (respectively, $x - y \in \mathbb{R}^p_-$) and $x \succ y$ (respectively, $x \prec y$) if and only if $x - y \in \mathbb{R}^p_{++}$ (respectively, $x - y \in \mathbb{R}^p_{--}$). In this paper, we consider the nonsmoothing multiobjective problem

$$P_{(1)} : \begin{cases} \min F(x), \\ \text{subjet to} \ \ x \in S. \end{cases}$$

where $S = \{x \in \mathbb{R}^n \ / \ g_j(x) \leq 0, \ j = 1, \cdots, m\}$, the objective function $F : \mathbb{R}^n \to \mathbb{R}^p$ is given by $F(x) = (f_1(x), \cdots, f_p(x))$ is nonsmooth, convexe and locally Lipschitz and $g_j : \mathbb{R}^n \to \mathbb{R}$ is continuously differentiable, $j = 1, \cdots, m$, S is a feasible set of $P_{(1)}$. The set of active indexes at a point $x \in S$ is given by $J(x) = \{j, \ g_j(x) = 0\}$. A point $x^* \in S$ is called a Pareto optimal point or (efficient solution) of problem $P_{(1)}$ if there exists no other $x \in S$ with $f_i(x) \leq f_i(x^*)$, $i = 1, \cdots, m$ and $f_j(x) < f_j(x^*)$ for at least one index j. If there exists no $x \in S$ with $f_i(x) < f_i(x^*)$ $i = 1, \cdots, p$, then

x^* is said to be a weak Pareto optimal point or (weak efficient solution) of problem $P_{(1)}$.

Definition 2.1. [4] The upper Clarke directional derivative of a locally Lipschitz function $f:\ \mathbb{R}^n \to \mathbb{R}$ at x in the direction $d \in \mathbb{R}^n$ is

$$f^{\circ}(x,d) = \limsup_{z\to x,\ t\downarrow 0} \frac{f(z+td) - f(y)}{t}$$

and the Clarke subdifferential of f at x is given by

$$\partial_c f(x) = \{\lambda \in \mathbb{R}^n :\ \ \langle \lambda, d\rangle \leq f^{\circ}(x,d)\ \ \forall d \in \mathbb{R}^n\}$$

When f is continuously differentiable, one has $\partial_c f(x) = \{\nabla f(x)\}$. Now, we recall some results which will be needed in our convergence analysis.

Proposition 2.1. *[4]*
Let $f:\ \mathbb{R}^n \to \mathbb{R}$ be locally Lipschitz and $h:\ \mathbb{R}^n \to \mathbb{R}$ continuously differentiable. Then

(i) $\partial_c(f(x) + h(x)) = \partial_c f(x) + \nabla h(x)$.

(ii) *If x^* is a local minimum of f, then $0 \in \partial_c f(x)$.*

(iii) *If $f(x) = \max\{f_1(x), \cdots, f_p(x)\}$, where $f_j:\ \mathbb{R}^n \to \mathbb{R}$ for all $j \in \{1,\cdots,p\}$ are continuously differentiable, then*

$$\partial_c f(x) = \mathbf{conv}\{\nabla f_j(x)\ :\ \ j = 1,\cdots,p \quad \text{such that } f_j(x) = f(x)\}$$

*(here **conv** denotes the convex hull).*

3 Smoothing function

Rockafellar and Wets have shown that for any locally Lipschitz function f, we can construct a smoothing function by using the convolution

$$f(x,\mu) = \int_{\mathbb{R}^n} f(x-y)\psi_\mu(y)dy = \int_{\mathbb{R}^n} f(y)\psi_\mu(x-y)dy$$

where $\psi_\mu:\ \mathbb{R}^n \to \mathbb{R}$ is a smooth kernel function, (see [14]). In this section, we extend the smoothing method given by Chen [3] to solve nonsmooth MOP. For this, we start by considering a class of smoothing functions.

Definition 3.1. Let $F:\ \mathbb{R}^n \to \mathbb{R}^p$ be a continuous function given by $F(x) = (f_1(x),\cdots,f_p(x))$, we define a smoothing function of F by $\widetilde{F}:\ \mathbb{R}^n \times \mathbb{R}^p_{++} \to \mathbb{R}^p$ where $\widetilde{F}(x,\mu) = (\widetilde{f}_1(x,\mu_1),\cdots,\widetilde{f}_p(x,\mu_p))$ such that for each $i = 1,\cdots,p$ $\widetilde{f}_i(x,\mu_i)$ is continuously differentiable in $\mathbb{R}^n$ for any fixed $\mu_i \in \mathbb{R}^p_{++}$, and for any $x \in \mathbb{R}^n$

$$\lim_{y\longrightarrow x,\mu_i\downarrow 0} \widetilde{f}_i(y,\mu_i) = f_i(x)$$

Now we can construct a smoothing method by using $\widetilde{F}$ and $\nabla\widetilde{F}$ as follows. The first step is to define a parametric smooth function $\widetilde{F}(x,\mu_k)$ to approximate $F(x)$. The second step is to find for a fixed $\mu_k \in \mathbb{R}^p_{++}$ an approximate solution of the smooth MOP

$$P_{(\mu_k)}: \begin{cases} \min \widetilde{F}(x,\mu_k), \\ \text{subjet to } x \in S. \end{cases}$$

In the last step, by updating μ_k, which guarantees the convergence of any accumulation point of a designated subsequence of the iteration sequence generated by the smoothing MOP algorithm is a AKKT point. So, the Pareto optimal solutions (stationary points) of the approximate subproblems $P_{(\mu_k)}$ converge to a Pareto optimal solution (stationary point) of the initial MOP $P_{(1)}$. Note that the advantage of the smoothing method is to solve optimization problems with continuously differentiable functions which has a rich theory and powerful methods [11].

Many nonsmooth optimization problems can be reformulated by using the plus function $(h)_+$, for exemple, $\max(h,g) = h + (g-h)_+$, $\min(h,g) = h - (h-g)_+$ and $|h| = (h)_+ + (-h)_+$. So that, in this paper, we present a class of smooth approximation for the plus function by the convolution given by Chen [3].

Definition 3.2. [3] Let $\rho: \mathbb{R} \to \mathbb{R}^+$ be a piecewise continuous density function satisfying

$$\rho(s) = \rho(-s) \quad \textit{and} \quad \kappa := \int_{\mathbb{R}} |s|\rho(s)ds < \infty$$

then, the function $\phi: \mathbb{R} \times \mathbb{R}^+ \to \mathbb{R}^+$ defined by

$$\phi(h,\mu) := \int_{\mathbb{R}} (t-\mu s)_+ \rho(s) ds \tag{3.1}$$

is a smoothing function of $(h)_+$.

Proposition 3.1. *[3]*
For any fixed $\mu > 0$, $\phi(\cdot,\mu)$ is continuously differentiable, convex, strictly increasing, and satisfies

$$0 < \phi(h,\mu) - (h)_+ \leq \kappa\mu \tag{3.2}$$

then for any $h \in \mathbb{R}$

$$\lim_{h_k \to h \quad \mu \downarrow 0} \phi(h_k,\mu) = (h)_+ \tag{3.3}$$

Proposition 3.2. *[3] Let $\partial(h)_+$ the Clarke subdifferential of $(h)_+$ and $G_\phi(h)$ is the subdifferential associated with the smoothing function ϕ at h given by*

$$G_\phi(h) = con\{\tau \ / \ \nabla_t \phi(h_k,\mu_k) \to \tau \ , \quad h_k \to h, \quad \mu_k \downarrow 0\} \tag{3.4}$$

then

$$G_\phi(h) = \partial(h)_+$$

Remark 3.3. The plus function $(h)_+$ is convex and globally Lipschitz continuous. Any smoothing function $\phi(h,\mu)$ of $(h)_+$ is also convex and globally Lipschitz. In addition, for any fixed h, the function ϕ is continuously differentiable, monotonically increasing and convex with respect to $\mu > 0$ and satisfies

$$0 \le \phi(t,\mu_2) - \phi(t,\mu_1) \le \kappa(\mu_2 - \mu_1) \qquad \text{for} \qquad \mu_2 > \mu_1$$

Now, we study properties of the smoothing function ϕ. We assume that $F : \mathbb{R}^n \to \mathbb{R}^p$ given by $F(x) = (f_1(x), \cdots, f_p(x))$ is locally Lipschitz continuous. According to Rademacher's theorem, F is differentiable almost everywhere. For each $i = 1, \cdots, p$ the Clarke subdifferential of f_i at a point x is defined by

$$\partial f_i(x) = conv\{v \ / \ \nabla f_i(z) \to v, \quad f_i \text{ is differentiable at } z, \quad z \to x\}$$

For a locally Lipschitz function f_i, the gradient consistency

$$\partial f_i(x) = conv\{\lim_{x_k \to x \quad \mu_k^i \downarrow 0} \nabla \widetilde{f}_i(x_k, \mu_k^i)\} = G_{\widetilde{f}_i}(x) \qquad \forall x \in \mathbb{R}^n$$

between the Clarke subdifferential and the subdiferential associated with the smoothing function of f_i for each $i = 1, \cdots, p$. Note that the above result is important for the convergence of smoothing methods.

Throughout the rest of this paper we assume that the function F is given by $F(x) = H((\varphi(x))_+)$ where $H(x)$ and $\varphi(x)$ are continuously differentiable, $H(x) = (h_1(x)\cdots, h_p(x))$ with components $h_i : \mathbb{R}^n \to \mathbb{R}, \quad i = \{1, \cdots, p\}$ and $\varphi(x) = (\varphi_1(x)\cdots, \varphi_n(x))$ with $\varphi_j : \mathbb{R}^n \to \mathbb{R}, \quad j = 1, \cdots, n$. Notice that $(\varphi(x))_+ = ((\varphi_1(x))_+, \cdots, (\varphi_n(x))_+)$ and its smoothing function is

$$\phi(\varphi(x),\mu) = (\phi(\varphi_1(x),\mu), \cdots, \phi(\varphi_n(x),\mu))^T.$$

Now we show the gradient consistency of the smoothing composite functions using ϕ in Definition 3.2 for the plus function.

Theorem 3.3. *Let $F(x) = H((\varphi(x))_+)$, where $\varphi : \mathbb{R}^n \to \mathbb{R}^n$ and $H : \mathbb{R}^n \to \mathbb{R}^p$ are continuously differentiable, then for each $i = 1, \cdots, p$, $\widetilde{f}_i(x,\mu_k^i) = h_i(\phi(\varphi(x)), \mu_k^i)$ is a smoothing function of f_i with the following properties.*

(i) *For any $x \in \mathbb{R}^n$, $\{\lim_{x_k \to x \quad \mu_k^i \downarrow 0} \nabla \widetilde{f}_i(x_k, \mu_k^i)\}$ is nonempty and bounded, and $\partial f_i(x) = G_{\widetilde{f}_i}(x)$, for each $i = \{1, \cdots, p\}$.*

(ii) *If H, φ_j are convex for each $j \in \{1, \cdots, n\}$ and φ_j is monotonically nondecreasing, then for any fixed $\mu_k^i \in \mathbb{R}^p_{++}$, $\widetilde{f}_i(\cdot, \mu_k^i)$ is convex.*

Proof. For any fixed $i \in \{1, \cdots, p\}$, we can derive this theorem by theorem 1 [3]. $\square$

Proposition 3.4. *Let $\vartheta(t) = |t|$, $\vartheta_\mu(t) = sin(\mu).ln(\cosh(\frac{t}{\sin(\mu)}))$, $0 < \mu < \frac{\pi}{2}$. Then*

(i) $0 \le \vartheta(t) - \vartheta_\mu(t) \le \sin(\mu)\ln(2)$.

(ii) $|\frac{d(\vartheta_\mu(t))}{dt}| < 1$, *and* $\frac{d(\vartheta_\mu(t))}{dt}|_{t=0} = 0$.

(iii) $\vartheta_\mu(t)$ *is convex.*

Proof. (i) We have

$$\begin{aligned}\vartheta_\mu(t) - \vartheta(t) &= sin(\mu).ln(\cosh(\frac{t}{\sin(\mu)})) - |t| \\ &= \sin(\mu).ln(\frac{1}{2}\exp(\frac{t}{\sin\mu}) + \frac{1}{2}\exp(\frac{-t}{\sin\mu})) - |t| \\ &= \sin(\mu).ln(\exp(\frac{t-|t|}{\sin\mu}) + \exp(\frac{-t-|t|}{\sin\mu})) + \sin(\mu).ln(\frac{1}{2})\end{aligned}$$

Since $1 < \exp(\frac{t-|t|}{\sin\mu}) + \exp(\frac{-t-|t|}{\sin\mu}) \leq 2$, thus,

$$\sin(\mu).ln(\tfrac{1}{2}) < \sin(\mu).ln(\exp(\tfrac{t-|t|}{\sin\mu}) + \exp(\tfrac{-t-|t|}{\sin\mu})) + \sin(\mu).\ln(\tfrac{1}{2}) \leq \sin(\mu).ln(2) + \sin(\mu).ln(\tfrac{1}{2})$$

and

$$\sin(\mu).\ln(\frac{1}{2}) < \vartheta_\mu(t) - \vartheta(t) \leq 0.$$

Hence, we obtain

$$0 \leq \vartheta(t) - \vartheta_\mu(t) \leq \sin(\mu)\ln(2).$$

Therefore, ϑ_μ is a smoothing approximation function ϑ.

(ii) We have

$$\frac{d\vartheta_\mu(t)}{dt} = \frac{\exp(\frac{t}{\sin\mu}) - \exp(\frac{-t}{\sin\mu})}{\exp(\frac{t}{\sin\mu}) + \exp(\frac{-t}{\sin\mu})} = \frac{\exp(\frac{2t}{\sin\mu}) - 1}{(\exp\frac{2t}{\sin\mu}) + 1}$$

Since

$$\left|\frac{\exp(\frac{2t}{\sin\mu}) - 1}{\exp(\frac{2t}{\sin\mu}) + 1}\right| < 1,$$

then

$$|\frac{d(\vartheta_\mu(t))}{dt}| < 1$$

(iii)

$$\begin{aligned}\vartheta_\mu''(t) &= \frac{d}{dt}\left[\frac{d(\vartheta_\mu(t))}{dt}\right] \\ &= \frac{\frac{4}{\sin\mu}\exp(\frac{2t}{\sin\mu}))}{(\exp(\frac{2t}{\sin\mu}) + 1)^2} > 0. \quad \forall \mu \in (0, \frac{\pi}{2}).\end{aligned}$$

Thus, ϑ_μ is convex.

□

Proposition 3.5. *Let $\vartheta(t) = |t|$, and a vector function $g(x) = (g_1(x), \cdots, g_p(x))^T$ with components $g_j : \mathbb{R}^n \to \mathbb{R}$, we denote $\vartheta(g(x)) = |g(x)| = (|g_1(x)|, \cdots, |g_p(x)|)$ and $\vartheta_\mu(g(x)) = (\vartheta_{\mu_1}(g_1(x)), \cdots, \vartheta_{\mu_p}(g_p(x)))^T$, with $\vartheta_\mu(g_j(x)) = sin(\mu_j).ln(\frac{1}{2}\exp(\frac{g_j(x)}{sin\mu_j}) + \frac{1}{2}\exp(\frac{-g_j(x)}{sin\mu_j}))$ for $j = \{1, \cdots, p\}$ and $0 < \mu_j < \frac{\pi}{2}$. Then $\vartheta_\mu(g(x))$ is a smoothing approximation function of $\vartheta(g(x))$.*

Proof. For any fixed $j \in \{1, \cdots, p\}$, we can derive by proposition 3.5 that

$$0 < \vartheta(g_j(x)) - \vartheta_{\mu_j}(g_j(x)) \leq \sin(\mu_j).ln(2).$$

Considering $\kappa = ln(2)$ and $0 \leq \sin(\mu_j) \leq \mu_j \ \forall \mu_j \in [0, \frac{\pi}{2}]$.
then,

$$0 < \vartheta(g_j(x)) - \vartheta_{\mu_j}(g_j(x)) \leq \kappa\mu \ \text{ for } \ j \in \{1, \cdots, p\}.$$

Therefore, $\vartheta_\mu(g(x))$ is a smoothing approximation function $\vartheta(g(x))$.

□

4 Smoothing multiobjective optimization problem

In this section, we introduce AKKT condition for the multiobjective problem $P_{(\mu)}$ inspired by Giorrg. G. et al. [6]. Then we exploit it to prove convergence analysis of the smoothing method, whose feasible set is defined by inequality constraints. In fact, the solution of problem $P_{(1)}$ is accomplished by solving a sequence of problems $P_{(\mu)}$, where the value of μ is updated according to $\mu_{k+1} = \alpha\mu_k$ with $\alpha \in (0, 1)$ is the decreasing factor of μ. We point out that Chen [3] is concerned with convergence analysis of the smoothing method (in the scalar case) by using a smoothing gradient method.

Definition 4.1. We say that the AKKT condition is satisfied for problem $P_{(\mu)}$ at a feasible point $x^* \in S$ if there exists a sequence $(x_k) \subset \mathbb{R}^n$ and $(\lambda_k, \beta_k) \subset \mathbb{R}^p \times \mathbb{R}^m$ such that

(C_0) $x_k \to x^*$

(C_1) $\sum_{i=1}^{p} \lambda_k^i \nabla \widetilde{f}_i(x_k, \mu_k^i) + \sum_{j=1}^{m} \beta_k^j \nabla g(x_k) \to 0$

(C_2) $\sum_{i=1}^{p} \lambda_k^i = 1$

(C_3) $g_j(x_k) < 0 \Rightarrow \beta_k^j = 0$ for $j = \{1, \cdots, n\}$.

Remark 4.2. (i) A point satisfying the AKKT is called an AKKT point.

(ii) The sequence of points (x_k) is not required to be feasible.

(iii) Assuming $\beta_k \in \mathbb{R}^m_+$, condition ($C_3$) is equivalent to

$$\beta_k^j g_j(x_k) \leq 0 \text{ for sufficiently large } k, \ \ \forall j \notin J(x^*).$$

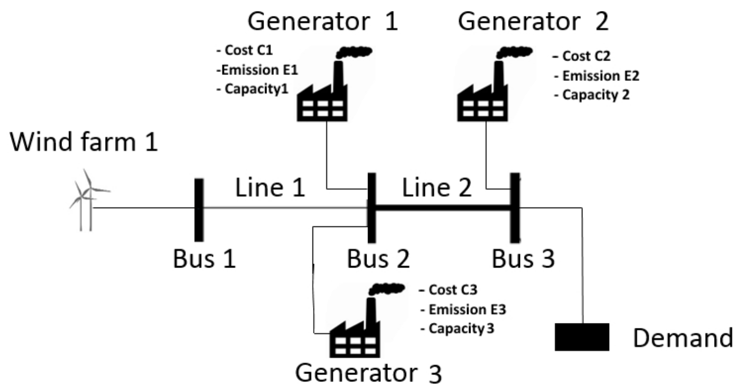

Figure 1: Bi-objective Non-smooth Environmental and Economic Dispatch Problem.

Each of these conditions implies the condition

$$\beta_k^j g(x_k) \to 0 \quad \forall j \notin J(x^*)$$

The following theorem establishes necessary optimality conditions for problem $P_{(1)}$.

Theorem 4.1. *If $x^* \in S$ is a locally weakly efficient solution for Problem $P_{(1)}$, then x^* satisfies the AKKT condition with sequences (x_k) and (λ_k, β_k). In addition, for these sequence we have that $\beta_k = b_k(g(x_k))_+$ where $b_k > 0 \quad$ for every k.*

Proof. The proof is based on Theorem 3.1 in [6], the gradient consistency, Theorem 3.3 and Proposition 3.1. □

In order to establish the sufficient condition, we assume the convexity assumption and the following condition.
Assumption A: We say that a sum is converging to zero, if $\sum_{j=1}^{m} \beta_k^j g(x_k) \to 0$.

Theorem 4.2. *Assume that H, φ_j for $j = \{1, \cdots, n\}$ and g_i for $i = \{1, \cdots, m\}$ are convex and φ_j is monotonically nondecreasing. If $x^* \in S$ satisfies the AKKT condition and assumption A is fulfilled, then, $x^*$is a global weak efficient solution of problem $P_{(1)}$.*

Proof. By Theorem 3.2 in [6] and the gradient consistency, Theorem 3.3, we abrove this Theorem. □

5 Bi-objective nonsmooth environmental and economic dispatch problem

The Bi-objective Economic and Environmental Dispatch Problem (EEDP) is concerned with the minimization of generation costs and the emission of pollutants while representing systems operational constraints. Note that the two objectives are conflicting in nature and they both have to be considered simultaneously to find the overall optimal dispatch. The EEDP is a multi-objective, nonlinear, and nonsmooth problem.

5.1 Notation

$C(P)$	cost function for all thermal units [$];
$E(P)$	total pollutant emission for all thermal units given in [kg/h];
P	vector of active power outputs for all thermal units [MW];
P_i	active power output of the generating unit i [MW];
E_i	active emission output of the generating unit i [MW];
$P_i^{\max}$	maximum power output of the generating unit i [MW];
$P_i^{\min}$	minimum power output of the generating unit i [MW];
$E_i^{\max}(P)$	maximum pollutant emission given in [kg/h];
$E_i^{\min}(P)$	minimum pollutant emission given in [kg/h].

5.2 Cost function

The growing costs of fuels and operations of power generating units require a development of optimization methods for Economic Dispatch (ED) problems. Standard optimization techniques such as direct search and gradient methods often fail to find global optimum solutions. The realistic operation of the ED problem considers the couple valve-point effects and multiple fuel options. The cost model integrates the valve-point loadings and the fuel changes in one frame. So, the nonsmooth cost function is given by [15]:

$$C(P) = \sum_{i=1}^{n} C_i(P_i) = \sum_{i=1}^{n} a_i P_i^2 + b_i P_i + c_i + \left| g_i sin(h_i(P_i^{min} - P_i)) \right|$$

such that : $P_i^{min} \leq P_i \leq P_i^{max}$
where g_i, h_i, a_i, b_i and c_i are the cost coefficients of the generator i.

5.3 Emission function

The emission function can be formulated as the sum of all types of emissions considered, with convenient pricing or weighting on each emitted pollutant. In this paper, only one type of emission NOx is taken into account without loss of generality [5]. The volume of NOx emission is given as a function of the generator output. That is, the sum of a quadratic and exponential function. The total amount of emission such as SO2 or NOx depends on the amount of power generated by unit. The NOx emission amount which is, the sum of a quadratic function can be realistically written as :

$$E(P) = \sum_{i=1}^{n} E_i(P_i) = \sum_{i=1}^{n} \alpha_i P_i^2 + \beta_i P_i + \gamma_i$$

where, α_i, β_i, γ_i, η_i and δ_i are the coefficients of the ith generator emission characteristics.
Formulation of the Non-Smooth EEDP:

$$P_{\text{NEEDP}}: \begin{cases} \min\{C(P), E(P)\} \\ \text{subjet to}: \ \sum_{i=1}^{n} P_i = P_d \\ P_i^{min} \leq P_i \leq P_i^{max} \end{cases}$$

5.4 Application:

In order to solve EEDP with two generators, we consider the following problem :

$$P_{\text{NEEDP}}: \begin{cases} \min\{C(P), E(P)\} \\ \text{subjet to}: \ P_1 + P_2 = 650 \\ 100 \leq P_1 \leq 600 \ \text{ and } 100 \leq P_2 \leq 400. \end{cases}$$

where
$C_1(P_1) = 0.001562P_1^2 + 7.92P_1 + 561 + |300\sin(0.0315(P^{min} - P_1))|$;
$C_2(P_2) = 0.00194P_2^2 + 7.85P_2 + 310 + |200\sin(0.042(P^{min} - P_2))|$;
$C(P) = C_1(P_1) + C_2(P_2)$;
$E_1(P_1) = 0.0126P_1^2 + 1.355P_1 + 22.983$;
$E_2(P_2)0.00765P_2^2 + 0.805P_2 + 363.70$;
$E(P) = E_1(P_1) + E_2(P_2)$.

Step 1: We apply the smoothing method to the nonsmooth objective function $C(P)$ (see Propositions 3.5 and 3.4) to obtain a smooth objective function $\tilde{C}(P,\mu) = \{\tilde{C}_1(P_1,\mu), \tilde{C}_2(P_2,\mu)\}$ as follows:

(5.1)
$$\begin{aligned}\tilde{C}_1(P_1,\mu) =& 0.001562P_1^2 + 7.92P_1 + 561 + \sin(\mu)\left[\ln\left(\frac{1}{2}\exp\left(\frac{200\sin(0.042(P^{min} - P_2))}{\sin(\mu)}\right)\right)\right] \\ &+ \frac{1}{2}\exp\left[\left(\frac{-200\sin(0.042(P^{min} - P_2))}{\sin(\mu)}\right)\right]\end{aligned}$$

(5.2)
$$\begin{aligned}\tilde{C}_2(P_2,\mu) =& 0.00194P_2^2 + 7.85P_2 + 310 + \sin(\mu)\left[\ln\left(\frac{1}{2}\exp\left(\frac{300\sin(0.0315(P^{min} - P_))}{\sin(\mu)}\right)\right)\right] \\ &+ \frac{1}{2}\exp\left(\frac{-300\sin(0.0315(P^{min} - P_1))}{\sin(\mu)}\right)\Bigg].\end{aligned}$$

Step 2: Each of P^{μ}_{NEEDP} subproblems has the form

$$P^{\mu}_{\text{NEEDP}}: \begin{cases} \min\{\widetilde{C}(P), E(P)\} \\ \text{subjet to}: \ P_1 + P_2 = 650 \\ 100 \leq P_1 \leq 600 \ \text{ and } 100 \leq P_2 \leq 400. \end{cases}$$

Step 3: The bi-objective subproblem P^{μ}_{NEEDP} is transformed into a set of single-objective subproblems using the ϵ-constraint method. For both methods, the objective function of the subproblems are smoothed by the smoothing method and the subproblems are solved by the interior point barrier method.

$$P^{\mu,\ \epsilon_l}_{\text{NEEDP}}: \begin{cases} \min \widetilde{C}(P, \mu), \\ \text{subject to } P_1 + P_2 = 650, \\ E(P) < \epsilon_l; \\ 100 \leq P_1 \leq 600, \quad 100 \leq P_2 \leq 400. \end{cases}$$

Step 4: To create a constraint bound vector, consider the number of Pareto points n = 70; let $\tau = \frac{E^{\max} - E^{\min}}{n}$ and $\epsilon_{l+1} = \epsilon_l + \tau$, $l = \{1, \cdots, n-1\}$ with $\epsilon_1 = E^{\min}$, and solve each smoothed single-objective subproblem $P^{\mu,\ \epsilon_l}_{\text{NEEDP}}$ by the interior point barrier method.

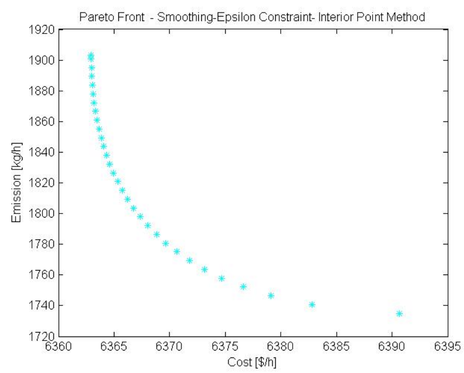

Figure 2: Pareto Front using Smoothing-$\epsilon-$constraint - Interior point method for $\mu =$ 0.000001.

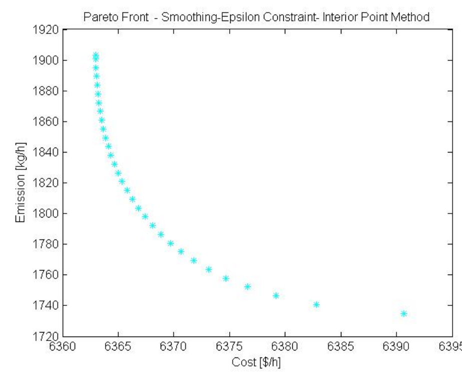

Figure 3: Pareto Front using Smoothing-$\epsilon-$constraint - Interior point method for $\mu =$ 0.0001.

In order to verify our approach performance, some simulations were performed and the results were compared with the PBC-HS-MLBIC method developed by Gonalves et al. [7]. This work was chosen because we use the same input parameters. In the following table, $\widehat{C}(P)$ and $\widehat{E}(P)$ represent respectively, the total cost and total pollutant emission obtained by PBC-HS-MLBIC method [7] for two generators.
From the results presented in table 1, some remarks are rised. The first is the quality of our methodology, as our output results are significantly better compared to that of Gonalves.

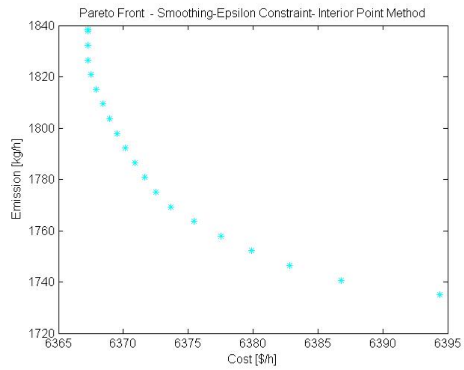

Figure 4: Pareto Front using Smoothing-$\epsilon-$constraint - Interior point method for $\mu =$ 0.01

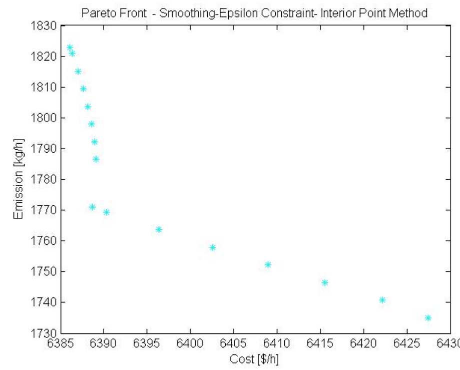

Figure 5: Pareto Front using Smoothing-$\epsilon-$constraint - Interior point method for $\mu =$ 0.1.

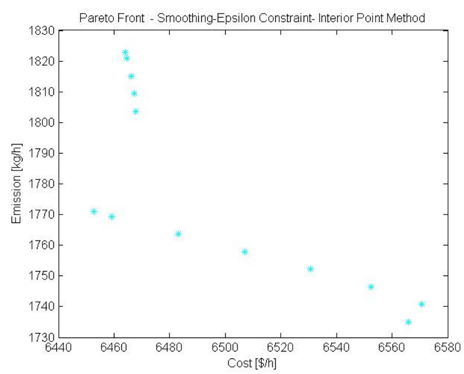

Figure 6: Pareto Front using Smoothing-$\epsilon-$constraint - Interior point method for $\mu =$ 0.5.

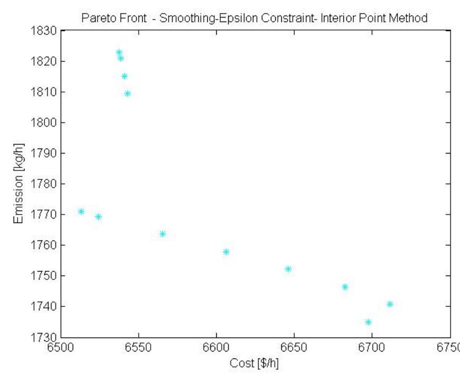

Figure 7: Pareto Front using Smoothing-$\epsilon-$constraint - Interior point method for $\mu =$ 1.

Conclusion.

In this paper, a class of nonsmooth multiobjective optimization problems that include min, max, absolute value functions or composition of the plus function $(t)_+$ with smooth functions is introduced, and some smoothing methods are presented. The algorithm is based on smoothing techniques to approximate the objective functions in all points where the function is nonsmooth. Numerical results show that the smoothing methods are promising for the nonsmooth MOP.

References

[1] E. G. Birgin, J. M. Martínez, *Fundamentals of Algorithms Practical Augmented Lagrangian Methods for Constrained Optimization*, vol 10. SIAM, Philadelphia, 2014.

Table 1: Minimum fuel cost, minimum emission and comparison with the output results of the PBC-HS-MLBIC method.

n	P_1(MW)	P_2(MW)	$C(P)$ (\$)	$E(P)$(kg/h)	$\widehat{C}(P)$ (\$)	$\widehat{E}(P)$(kg/h)
1	259,0835	390,7781	6390,6884	1735,0267	6698,45	1737,14
2	274,8133	375,1867	6382,8279	1740,72	6724,26	1747,20
3	282,1196	367,8804	6379,16	1746,44	6681,07	1752,92
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
35	350,0203	299,9797	6362,9941	1903,0076	6848,11	1936,07
36	350,0236	299,9764	6362,9941	1903,0198	6845,91	1941,79
37	350,0243	299,9757	6362,9941	1903,0223	6842,53	1947,51
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
68	350,0243	299,9757	6362,9941	1903,0225	6407,48	2124,93
69	350,0243	299,9757	6362,9941	1903,0223	6389,74	2130,66
70	350,0242	299,9758	6362,9941	1903,0222	6383,05	2135,34

[2] L. Chen D. Goldfarb, *Interior-point 2-penalty methods for nonlinear programming with strong global convergence properties*, Math Program. 108 (2006), 1-36.

[3] X. Chen, *Smoothing methods for nonsmooth, nonconvex minimization*, Math. Program. 134 (2012), 71-99.

[4] F. B. Clarke, *Optimization and Nonsmooth Analysis*, Wiley, New York, 1983.

[5] A. Farag, S. Al-Baiyat, T. C. Cheng, *Economic load dispatch multiobjective optimization procedures using linear programming techniques*, IEEE Transactions on Power Systems, 10, 2 (1995), 731-738.

[6] G. Giorgi, B. Jimenez, V. Novo, *Approximate Karush-Kuhn-Tucker condition in multiobjective optimization*, J Optim Theory Appl. 171 (2016), 70-89.

[7] E. Gonsalves, A. R. Balbo, D. N. da Silva, L. Nepomuceno, E. C. Baptista, E. M. Soler, *Deterministic approach for solving multi-objective non-smooth environmental and economic dispatch problem*, International Journal of Electrical Power and Energy Systems, 104 (2019), 880-897.

[8] E. S. Mistakidis, G. E. Stavroulakis, *Nonconvex optimization in mechanics. Smooth and nonsmooth algorithms*, Heuristics and Engineering Applications by the F.E.M., Kluwer Academic Publisher, Dordrecht, 1998.

[9] J. J. Moreau, P. D. Panagiotopoulos, G. Strang (Eds), *Topics in Nonsmooth Mechanics*, Birkhäuser, Basel, 1988.

[10] K. Miettinen, *Nonlinear Multiobjective Optimization*, Kluwer Academic Publishers, Boston, 1999.

[11] J. Nocedal, S. J. Wright, *Numerical Optimization*, 2nd edn., Springer, New York, 2006.

[12] J. Outrata, M. Kôcvara, J. Zowe, *Nonsmooth Approach to Optimization Problems with Equilibrium Constraints. Theory, Applications and Numerical Results*, Kluwer Academic Publishers, Dordrecht, 1998.

[13] L. Qi, Z. Wei, *On the constant positive linear dependence condition and its application to SQP methods*, SIAM J. Optim. 10 (2000), 963-981.

[14] R. T. Rockafellar, R. J.-B. Wets, *Variational Analysis*, Springer, New York, 1998.

[15] J. Zhan, Q. H. Wu, C. Guo, X. Zhou, *Economic dispatch with non-smooth objectives- part ii: dimensional steepest decline method*, IEEE Trans. Power Syst. 30, 2 (2015), 722-733.

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