

# Gheorghe Tzitzeica - Romanian scientist of international recognition

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## Abstract

Are presented the personality, the educational and scientific activity of the great Romanian scientist Gheorghe Tzitzeica.

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**Key words:** history, biography.

The personality, the educational and scientific activity of the great Romanian scientist Gheorghe Tzitzeica is presented hereby.

"There are several ways to pay homage to great departed scientists. Sometimes the impetuous wind of progress erases their footsteps. Not forgetting them is to continue their work, linking them organically to the living present." (Octav Mayer)

Gheorghe Tzitzeica was born October 4-th, 1873 in Turnu-Severin. He graduated the elementary school in Turnu-Severin and secondary school at "Carol I" High-school in Craiova. In 1882 he started to attend the Bucharest University, Mathematics division, having as professor, amongst others, Spiru Haret.

In 1895 he received a graduation diploma in mathematics and in 1896 he leaves for Paris where he graduates "École Normale Supérieure" also in mathematics, ranking the first amongst all graduates, French or foreign. In Paris, Tzitzeica had the opportunity to attend the classes of the famous mathematicians: Darboux, Picard, Poincaré, Appel, Goursat, Hadamard, Borel. Just mentioning these famous names, one can realize the premises under which Gheorghe Tzitzeica developed his personality both as a scientist and as a cultural figure. Not only that, in the capital of France he made himself many friends as well as spiritual connections that proved helpful later on in his work to appraise the fame of the Romanian science community.

At "École Normale Supérieure" he was both a year mate and friend to Lebesgue and Paul Montel. The close relationship between Tzitzeica and Lebesgue was based on their common passion for geometry, to whom many of their talks were dedicated. The friendship between Lebesgue (a humble, lonely and quiet human being) and Tzitzeica (always vivid and happy yet doubled by a great diligence) was, of course, conform to the profound affinity, revealed in a letter sent by Lebesgue to Octav Onicescu after Tzitzeica's departure. Lebesgue tried, in the above-mentioned document

to describe in few words the remarkable personality that the Romanian mathematician was. Lebesgue wrote: "I was thrilled to find him again happy, vivid, delighted to talk to me about his home, with that magnificent moral health radiating from his luminous yet thorough look in his eyes. (...) I understood then, that inside himself, laid an everlasting union between the sense of the duty to be achieved and the euphoria rising from the conscience of the fulfilled duty (...) and I discovered that our friendship for him was always shaded by an even greater respect". After graduating in Paris,

Gheorghe Tzitzeica prepared his doctoral degree, held in front of the examining committee chaired by Darboux himself, on June 30-th, 1899 at Sorbonne.

Returning to Romania, Gheorghe Tzitzeica was appointed assistant professor at the University of Bucharest and he got in charge of teaching the Differential and Integral Calculation course. On May 4-th, 1900, Gheorghe Tzitzeica was appointed professor of Analytical Geometry, position that he held until February 5-th, 1939, the date of his premature departure. Between 1928 and 1939 he was also professor at the Mathematical analysis division of the Polytechnic School.

Internationally well-known scientist, Gheorghe Tzitzeica was one of the most prodigious pedagogues of the Romanian education. At the University, Tzitzeica taught analytical geometry to first-year students as well as, since 1912, superior geometry to third-year students. Whilst changing every year his expos, Tzitzeica dealt regularly in the first year with the ordinary analytical bi-dimensional and tri-dimensional geometry. Ever since 1912, Tzitzeica brought to the third-year students (the final-year students) a special course that dealt, whilst the subject was changed every year, with a chapter of analytical geometry that was a development to the general course. Thus, he lectured on mapping, constant curvature surfaces, conjugated webs theory, ruled geometry, local geometry, ruled surfaces, webs theory, metrical properties of the space, minimal surfaces, Weingarten congruencies, geometrical calculation, mobile reference point (Darboux) method, groups theory, conformal representation, conformal geometry and movement geometry.

Tzitzeica pled essential services to the educational system throughout his exercise books of elementary and analytical geometry whilst his brochure on projective differential geometry of the curves as well as his book on projective differential geometry of the webs are fundamental for every research project.

It would be advisable to say that, throughout the years, the exercises in Tzitzeica's exercise book were a source of inspiration for the authors of school manuals. Furthermore, the successes of our students at the international mathematics Olympic contest in the past 40 years are due to their excellent training in geometry, training credited mainly to Tzitzeica's exercise book.

Living proof of his devotion to the matters related to high-school education, Gheorghe Tzitzeica taught between 1936 and 1937 one-semester facultative courses on the methodic of teaching mathematics.

The teaching method of Tzitzeica is of a particularly perfect pedagogical art. Here is what an old student of Tzitzeica wrote (professor N.N. Mihaileanu, Ph.D.D.):" After Tzitzeica's courses one would have left home bearing the teaching in his mind. But this is an understatement. It is hard to express in words the internal harmony of Tzitzeica's courses. Each course left you with a strong feeling of delightfulness; the one you would have expresses when being confronted with a painting. One would have

left the courses of this apostle of geometry abiding by both his example of dignity and straightness that he was for his entire life and by the optimistic belief the mathematics, in general, and, especially, geometry have a high and admirable educational value for young people.”

Many high-school teachers attended last-year courses, besides students. Through Tzitzeica’s and Pompeiu’s courses a permanent link between the faculty and the high-school teachers was kept alive. Each one of them was a master. If you had understood from Pompeiu that mathematics is both simple and beautiful, then it would have been Tzitzeica who would have taught you how to teach.

The great support that Tzitzeica gave to the pre-university education can also be shown by his relentless activity for the Journal of Mathematics, whose main publisher he was and where he published 34 papers (on arithmetic, algebra, elementary geometry, analytical geometry, trigonometry and history of mathematics), 18 notes and suggested 121 exercises.

Together with professor Longinescu, he founded in 1905, the ”Natura” magazine, a popular science journal.

In connection to the activity performed at each of the above-mentioned magazines and also to the fact that Tzitzeica changed yearly his expos, Dan Barbilian (former assistant professor to Tzitzeica) wrote: ” This man’s life is split between the faculty, where his Analytical Geometry course flows like a river of clarity whose waters can not be seen twice, the two magazines and his scientific work. Unlike ours, his life passes, aside from worries, equally and exemplary.”

I will briefly refer to some aspects of the mathematical works of Gheorghe Tzitzeica.

The first important aspect refers to the webs and congruencies theory. Most of the research on this aspect is detailed in his book ”Differential projective geometry of the webs” (1923). It is the guidebook, still valid and unanimously appreciated. The Chinese scientist Buchin Su translated the book into Chinese. Professor Gh. T. Gheorghiu, PhD wrote: ”Those who read this book are impressed by the coherence of the results as well as by the intuition that led the research. It is a marvelous example of scientific research”. The book published in 1923 makes a holistic, unitary and complete synthesis of the subject, paving the way and surging future researches. Amongst those who used and developed Tzitzeica’s ideas one can quote Bompiani, Calapso, Godeaux, Finikov, A. Pantazi, Gh. T. Gheorghiu, T. Mihailescu, O.Tino.

The second important aspect in Tzitzeica’s works is represented by the Tzitzeica curves and surfaces. Tzitzeica surfaces were developed in 1907 and are characterized by the constant ratio  $\frac{K}{d^4}$ , where  $K$  is the total curvature for a certain point  $M$  and  $d$  is the distance from a fixed point (center of the surface) to the tangent plane at the surface through point  $M$ .

Tzitzeica curves were developed in 1911 and are characterized by  $\frac{T}{d^2} = \text{constant}$ , where  $T$  is the torsion in a certain point  $M$  of the curve and  $d$  is the distance from a fixed point to the osculatory plane in  $M$ . This aspect was the subject of further research by Wilczynski, Blaschke, Al. Myller, O. Myer, Gh. T. Gheorghiu, Calugareanu, V. Cruceanu, Gh. Gheorghiev, I. Popa, I. Teodorescu, M. Putinar, G. Pripoe. C. Udriste, N. Bâla. In 1967, A. Dobrescu developed the Tzitzeica connection, offering to the Tzitzeica hypersurfaces a series of new properties. In 1969, K. Yano extended Dobrescu’s research referring to the Tzitzeica connection. In 1972, M. Tarina established

the link between Tzitzeica connection (in Dobrescu and K. Yano's understanding) and the projective and sub-projective connections.

The full value of Tzitzeica's works is far from being realized, however, as the ideas of the great Romanian scientist go beyond the strict framework of mathematics. Thus, in 1973, K. Teleman indicated the way to use some of Tzitzeica's principles to get symmetry models within the framework of elementary particles physics. Amongst the members of the Geometry division of the Faculty of Mathematics and Informatics (University of Bucharest) one may quote, from those that had researches linked to Tzitzeica's works, the following: Gh. Vranceanu, K. Teleman, A. Dobrescu, I. Teodorescu, S. Ianus, G. Pripoe and A.M. Teleman.

Gh. Vranceanu said, at a conference held at the Faculty of Mathematics in 1979, that "Gh. Tzitzeica discovered ever since 1908 the notion of space with affine connection, 10 years prior to H. Weyl". Establishing the priority of the Romanian scientist, professor Gh. Vranceanu concludes: "... not only Tzitzeica introduced for the first time the surfaces with affine connection but he also raised the subject of the embedding of such surfaces. In Weyl's work such embedding problems do not exist as they do not exist in any of the geometries with projective connection considered by Cartan".

Resuming what we have said so far, we underline that it is Gh. Tzitzeica who is credited of founding the Romanian School of Differential Geometry. His work provided fundamental and original results that proved to be as many openings for future research. Amongst those results one may quote:

- \* Contributions to the development of the geometries with fundamental group and the creation of differential central affine geometry;

- \* Definition and studies on the webs, congruencies, curves and surfaces bearing his name;

- \* Definition of connection, 10 years prior to Weyl.

We have mentioned just a few of the pioneer ideas of Tzitzeica. The reason for that is not only the impossibility of synthesizing them in such a short amount of space but also because his scientific work has still many hidden treasures. These treasures are yet insufficiently valued and await a new rise once they are rediscovered and set on the pedestal they deserve.

The activity of Gh. Tzitzeica was highly appreciated both inside the country and abroad. In 1931 he got elected as fellow of the Romanian Academy, later on becoming its vice-president as well as general secretary.

Due to his works on differential geometry, published in several mathematics journals, Tzitzeica became, as time grew by, both well known and appreciated in the mathematics world. Thus, at the International Congresses in Toronto (1924), Zurich (1932) and Oslo (1936) he was elected president of the geometry section, this being the greatest honor our great mathematician could plead to his country.

In 1926, 1930 and 1937 he taught as associated professor to Sorbonne. For example, in 1930, he taught a special course on webs and congruencies. He also taught courses at the universities in Brussels (1926) and Rome (1937). The two mathematicians Montel and Arnaud Denjoy wrote about Tzitzeica's and Pompeiu's courses at Sorbonne: "The French university is honored to have amongst its professors the Romanian scientists Tzitzeica and Pompeiu".

In 1930, Gheorghe Tzitzeica was elected correspondent fellow of the Maryland Academy of Science. In 1934 he became fellow of the Royal Society of Science in Liège and, in the same year, he is declared Doctor honoris causa of the University of Warsaw, his name becoming thus well known in the entire world.

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