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## Abstracts [26 August 2016]

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### Invariant, anti-invariant and slant submanifolds of a para-Kenmotsu manifold

*Adara Monica Blaga*

**Short abstract.** Properties of an invariant, anti-invariant and timelike-slant isometrically immersed submanifold  $M$  of a para-Kenmotsu manifold are given. We prove that if  $M$  is invariant, then it is para-Kenmotsu, as well.

### Family of groups on Descartes Folium. I

*Adrian Constantinescu, Steluta Pricopie, Constantin Udriște*

**Abstract.** The Descartes Folium stores natural group structures discovered by our research team. One of the main questions is to put in light their networking. In [2] we introduced the notion of  $\mathbf{K}$ -group as well as some of its properties. In this talk we will consider  $\mathbf{K}=\mathbf{R}$  (whose algebraic closure is  $\mathbf{C}$ ) and the projective Descartes Folium  $DF$  over  $\mathbf{R}$ . Firstly we will define two natural affine smooth algebraic  $\mathbf{C}$ -varieties, obtained by the modification of the algebraic  $\mathbf{C}$ -variety structure of the projective Descartes Folium over  $\mathbf{C}$  in its unique non-smooth point (there exists a natural relation between their underlying complex topologies and the analytic  $\mathbf{C}$ -manifolds associated to these algebraic  $\mathbf{C}$ -varieties with those discussed in [3]). The goal of this talk is to describe all  $\mathbf{R}$ -groups on  $DF$  w.r.t. each of these affine smooth algebraic  $\mathbf{C}$ -varieties.

#### References

- [1] A. Constantinescu, C. Udriște, S. Pricopie, *Genome of Descartes Folium via normalization*, manuscript.
- [2] A. Constantinescu, C. Udriște, S. Pricopie, *Classic and special Lie groups on some plane cubic curves with singularities. II*, to appear in Rev. Roum. Math. Pures et Appl.
- [3] C. Udriște, A. Constantinescu, S. Pricopie, *Topology and differential structure on Descartes Folium*, Ann. Sofia Univ., Fac. Math and Inf., 103 (2016), 1-9.

### Motion of charged particle in magnetic field created by electrical circuits

*Adriana Calotă, Nona Crăciun*

**Short abstract.** We compare two motions of a charged particle in magnetic field environment: (i) movement governed by Lorentz Law and (ii) movement as geometric dynamics.

### Completion of the group arising from fractional calculus,

*Akira Asada*

**Short abstract.** Considering the free Abelian group  $A_{\mathbf{R}}$  generated by fractional Euler differentiation the paper examines the completion of the free Abelian group  $A^{\#}_{\mathbf{R}}$  generated by  $A_{\mathbf{R}}$  and the extended Borel transformations.

### Second order tangent bundles and Geometric Mechanics - an alternative approach

*Ali Suri*

**Abstract.** The aim of this paper is to geometrize time - dependent Lagrangian mechanics in a way that the framework of second order tangent bundles plays an essential role. To this end, we first introduce the concepts of time dependent connections and time dependent semi-sprays on a manifold  $M$  and their induced vector bundle structures on the second order time dependent tangent bundle  $\mathbf{R} \times T^2M$ . Then we turn our attention to regular time Lagrangians and their interaction with  $\mathbf{R} \times T^2M$  in different situations such as mechanical systems with potential fields, external forces and holonomic constraints. Finally, we propose an example to support our theory.

### Riemannian optimal control

*Andreea Bejenaru, Constantin Udriște*

**Abstract.** This paper starts by expressing the general multi-time maximum principles corresponding to optimal control problems with various types of cost functionals; there are analyzed several situations: multiple integral-type, hyper-surface integral-type, curvilinear integral type and, also  $p$ -dimensional integral-type cost functionals. Moreover, when considering evolution constraints having symmetries, some adapted symmetric optimality PDEs are derived. The second part of the paper adapts the optimal control theory to a Riemannian setting. More precisely, the constraints are considered to be the second order metric compatibility evolution PDEs; the evolution ("multi-time") variables are the

local coordinates on a Riemannian manifold, the state variables are the Riemannian structure and the corresponding Levi-Civita linear connection, while the control variable is the Riemann curvature tensor field compatible with the Riemannian metric. Therefore, the intention is to find necessary conditions for the Riemannian metric in order to optimize a given functional. An interesting fact is that curvilinear integrals may be constrained only by first order metric compatibility evolution system.

### **Curvature inheritance symmetry**

*Chayan Kumar Mishra*

**Short abstract.** In this paper we study Curvature inheritance symmetry and Ricci - inheriting symmetry in Finsler space and investigated some results.

### **Barrier algorithm for optimization on manifolds with affine connections**

*Constantin Udriște, Ali Sapeeh Rasheed, Ionel Tevy*

**Short abstract.** The main aim of this paper is twofold: (i) to underline the role of the Hessian semi-Riemannian structure in the programming problems, and (ii) to find a suitable criterion of stopping in a Hessian barrier descent algorithm.

### **Evolution of Tzitzeica hypersurfaces**

*Constantin Udriște, Ionel Tevy, Saad Abbas Abed, Ali Sapeeh Rasheed*

**Abstract.** Our aim is to study the evolutions of Tzitzeica hypersurfaces. Sections 1-5 refer to standard Tzitzeica hypersurfaces and their evolutions: (i) evolution along the normal vector field, (ii) infinitesimal normal transformation of Tzitzeica hypersurface, (iii) evolution along a centro-affine vector field. Section 6 gives an affine diffeomorphism that preserve the quality of Tzitzeica hypersurface. Section 7 refers to parametric Tzitzeica surfaces as solutions of a PDE and their evolutions.

### **Geometric programming approaches of reliability allocation**

*Constantin Udriște, Saad Abbas Abed, Ionel Tevy*

**Abstract.** One of the important problems in the reliability design of a system is to allocate the reliability values to diverse constitutive units of the system. Every system has a reliability goal that needs to be achieved. Reliability allocations are used to set the goals for various subsystem or functional blocks such that the overall system level reliability can be achieved in an effective way. Our model discussed the posynomial cost function, taking into account all its properties regarding multivariate monotony and convexity (either Euclidean or with respect to a connection). Such a cost and the reliability constraint, associated to reliability polynomial, lead us to geometric programming method.

### **Some remarks on the product of Golden Structure**

*Cristina Elena Hretcanu*

**Short abstract.** In this paper we define the product of two Golden Riemannian manifolds and we find some properties of the Riemannian curvature tensor. Some remarks regarding the local symmetry and flatness for the product of two Golden Riemannian manifolds, are given.

### **On some results using measures of noncompactness,**

*Eberhard Malkowsky*

**Abstract.** We review some results on measures of non-compactness, including an axiomatic introduction, a study of the Kuratowski, Hausdorff, inner Hausdorff and separation measures of non-compactness, a survey of their most important properties and their roles in fixed point theory and operator theory. We also determine the Hausdorff measure of non-compactness of matrix operators and, more generally, of bounded linear operators between some BK spaces. Among other things we present many of our results (with co-authors) involving the classical sequence spaces, matrix domains of triangles in certain sequence spaces, and spaces of strongly summable and bounded sequences.

### **On the discrete version of Linear Quadratic Games**

*Elena Laura Otobîcu, Andreea Novăcescu*

**Short abstract.** The purpose of this paper is to study the discrete version of Linear Quadratic Games on the time scale. It contains some preliminaries regarding the time scale, the optimization of linear systems on time scale, the linear quadratic regulator with fixed final state and several illustrative examples.

## Classification of contact structures on the complex indicatrix

*Elena Popovici*

**Abstract.** By considering the complex indicatrix as an embedded CR-hypersurface of the holomorphic tangent bundle in a fixed point, we discuss some aspects of the relation of its CR structure and the contact structure introduced on it. Moreover, having in mind the classification of the almost contact metric structures associated with a strongly pseudo-convex CR-structure, of D. Chinea and C. Gonzales, we find the classes corresponding to the natural contact structure of the complex indicatrix and the new structure obtained under a gauge transformation.

## On some solvability aspects for the Schroedinger and Dirac equations in 2+1 and 3+1 dimensional spaces

*George Krylov*

**Short abstract.** Some classes of exactly and quasi-exactly solvable problems for the Schroedinger and Dirac operators are considered. It has been demonstrated that in 2+1 dimensional space case the number of integrable quantum systems for the Dirac equation is larger than that for the 3+1 case.

## Pseudo-Finsler geometry modeling of Langmuir monolayer with edge

*Halina Grushevskaya, Nina Krylova, George Krylov*

**Abstract.** Two-dimensional (2D) motion of a particle under the action of electrocapillary forces in a Langmuir monomolecular layer (monolayer) with edge has been analysed with the help of Finsler geometry methods. A spherically symmetric pseudo-Finsler 2D-billiard has been constructed to describe the influence of boundary on dynamics of the particles in the Langmuir monolayer with edge. Finsler reflection law has been determined. The dynamics of the particle has been studied in case of pseudo-Finsler (2+1) metric function being approximated by pseudo-Riemannian metric with small Finsler corrections. The pseudo-Finsler corrections are revealed as apparent dependence of particle mass on its velocity and coordinates. The angle of reflection becomes larger than angle of incidence at high monolayer compression rates.

## Some properties of concircular transformations of Kahler manifolds

*Iulia Elena Hirićă*

**Abstract.** We introduce the notion of concircular holomorphically projective mappings between Kahler manifolds, along the line of conformal geodesic mappings between Riemannian manifolds and conformally holomorphically projective mappings on Kahler spaces. Using properties of the  $J$ -traceless component of the Weyl conformal tensor  $W$ , we obtain a rigidity condition such that a concircular  $HP$ -mapping is a concircular one. Also, some families of concircular metrics are built, considering techniques of certain  $W$ -Kahler-Riemann type flows.

## Complete integrability of geodesics in toric Sasaki-Einstein space $T^{1,1}$ and action-angle variables

*Mihai Vişinescu*

**Abstract.** We describe the construction of the integrals of geodesic motions in the homogeneous Sasaki-Einstein space  $T^{1,1}$ . For this purpose we use the knowledge of the complete set of Killing vectors and Killing tensors of this space. We discuss the integrability of geodesics and construct explicitly the action-angle variables. We find that two pairs of frequencies of the geodesic motions are resonant giving way to chaotic behavior when the integrable Hamiltonian is perturbed by a small non-integrable piece.

## Riemann-Lagrange geometric dynamics for the multi-time magnetized non-viscous plasma

*Mircea Neagu*

**Abstract.** In this paper, using Riemann-Lagrange geometrical methods, we construct a geometrical model on 1-jet spaces for the study of multi-time relativistic magnetized non-viscous plasma, characterized by a given energy-stress momentum distinguished ( $d$ -) tensor. In that arena, we give the conservation laws and the continuity equations for multi-time plasma. The partial differential equations of the stream sheets (the equivalent of stream lines in the classical semi-Riemannian geometrical approach of plasma) for multi-time plasma are also written.

## On the set of optimal initial conditions in the Dai-Schlesinger's algorithm

*Nina Aprausheva, Vasily Dikusar, Stanislav Sorokin*

**Abstract.** The Dai-Schlesinger algorithm - often called EM-algorithm in the scientific literature - is designed to calculate the unknown parameters of the Gaussian mixture available sample. To determine the parameters of the mixture, there are used well-known classical methods, modification of these methods and implementation programs. To calculate

the optimal parameter estimate for multidimensional blending, the most effective and commonly used algorithm is the Dai-Schlesinger one, based on two methods: maximum likelihood and Picard successive approximations. This algorithm is used in various fields of science and practice of the Gaussian mixture model with equal covariance matrices and vectors with different mean values, due to its resistance to violations of the assumptions of normality and completeness of the system of Gaussian functions in the space  $L_2(-\infty, \infty)$ .

In several studies with reference to the source, it was noted that the probability  $P$  optimal solutions for the algorithm Dai-Schlesinger at random initial conditions dramatically decreases for increasing dimension  $p$  of the sample space. However, it was found experimentally that the probability  $P$  is a decreasing function of the parameters  $p, k, \mathcal{E}$ , (where  $k$  is the number of components of the mixture and  $\mathcal{E}$  is the accuracy of calculations), and an increasing function of the parameters  $\rho_{is}, n$  (where  $\rho_{is}$  is the Mahalanobis distance between the components of the mixture,  $i < s, i, s \in \{1, 2, \dots, k\}$  and  $n$  is the sample size). This should be confirmed by means of experimental analysis.

When  $k \geq 2$ , for any dimension  $p$  of the sample space, there exist  $\rho_0 = \rho(k, p), n_0 = n(k, p)$  and  $\varepsilon_0$  ( $0 < \varepsilon_0 < 10^{-8}$ ) such that, for all  $\rho_{is} \geq \rho_0, n \geq n_0, 0 < \varepsilon < \varepsilon_0$  we get the optimal solution  $P \geq 0.5$ .

In addition, this study adjusts the rule for choosing the optimal solution, which takes into account the frequency of each solution at random initial conditions.

### **Elastic strips with pseudo-null directrix**

*Ozkan Tükel G., A. Yücesan*

**Abstract.** In this work, we firstly determine equilibrium equations for elastic strips with pseudo-null directrix and classify the pseudo-null directrix by solving these equations. Then, we show that the tangent of a solution pseudo-null curve of the equilibrium equations corresponds a null elastic curve in De Sitter 2-space and the binormal of this solution pseudo-null curve corresponds a spacelike elastic curve in 2-dimensional light-like cone. Furthermore, we derive two conservation laws for elastic strips with pseudo-null directrix.

#### **References**

- [1] Chubelaschwili D., Pinkall U., *Elastic strips*, Manuscripta Mathematica, 133 (2010), 307-326.
- [2] Grbovic M., Ilarslan K., Nesovic E., *On null and pseudo null Mannheim curves in Minkowski 3-space*, J. of Geometry, 105, 1 (2013), 177-183.
- [3] Ilarslan K., Yildirim M., *An application of Ritt-Wu's zero decomposition algorithm to the pseudo null Bertrand type curves in Minkowski 3-space*, J. Systems Science and Complexity, 24, 2 (2011), 358-366.
- [4] Liu H., *Curves in the Lightlike Cone*, *Beiträge zur Algebra und Geometrie*, Contributions to Algebra and Geometry, 44, 1 (2004), 291-303.
- [5] Tükel Özkan G., *Elastic strips in Minkowski 3-space*, Suleyman Demirel, University Graduate School of Natural and Applied Science, PhD Thesis, Isparta, 2014.
- [6] Tükel Özkan G., Yücesan A., *Elastic curves in a two-dimensional lightlike cone*, Int. Electronic J. Geom., 8, 2 (2015), 1-8.

### **Busemann G-spaces with non-positive curvature as singular Finsler spaces**

*Pavel Andreev*

**Short abstract.** It is shown that the tangent cone to Busemann  $G$ -space  $X$  with non-positive curvature has a structure of strictly convex normed space. Consequently, the space  $X$  may be considered as singular Finsler space. Some properties of the space  $X$  which are natural in Finsler geometry are generalized.

### **Discrete recurrences and Floquet Theory in multi-time framework (PhD Thesis - summary)**

*Raluca Tuligă (Coadă)*

**Abstract.** This Thesis introduces and discusses multi-time recurrences: discrete multi-time diagonal recurrence, discrete multi-time multiple recurrence, discrete multi-time recurrence of way required. They arise in the analysis of algorithms, computational biology, information theory, theory of waiting arrays, filters theory, statistical physics, etc., by particular cases. For the recurrences mentioned above, we find solutions, transition matrix of states, we clarify structural context, we offer conceptual proofs and present relevant applications. We also extend the Floquet theory to multi-time diagonal recurrences.

### **Stochastic accessibility along posynomial distributions**

*Teodor Țurcanu, Constantin Udriște*

**Short abstract.** We prove an accessibility Theorem via admissible stochastic processes induced by stochastically perturbed posynomial type distributions.

## On the inertia of quadratic self-adjoint matrix polynomials

*Valeriu Prepeliță, Tiberiu Vasilache, Mona Doroftei*

**Abstract.** The concepts of the spectral theory for self-adjoint matrix polynomials are applied to a class of quadratic matrix polynomials to obtain a direct and simple description of their inertia. Two formulas for the signature and the rank are derived. These results have applications in various domains such as systems and control theory. An example illustrates the behavior of the proposed methods for self-adjoint matrix polynomials of order two.

## About the minimum of a maximal heating for the reentry body; 3. optimal control problems,

*Vasily Dikusar, Marian Koska, Adam Figura, Marek Wojtowicz*

**Abstract.** We consider the problem of choice of an angle of attack by a device which is slowed down in the atmosphere for the flight on the minimum of maximum heat with constraints on the value of full loading factor. The solution of mentioned problems allows to determine maneuver abilities of the device. The heat of the device flight is determined by the following integral

$$Q = \int_0^T C \rho^{1/2} V^3 dt. \quad (1)$$

We aim to determine the control  $C_y(t)$  such as to minimize  $Q(t)$  given by (1), under the following constraints:

$$\begin{aligned} n_{\Sigma} &= \sqrt{C_x^2 + C_y^2} q \frac{S}{G} \leq N, & q &= \frac{\rho V^2}{2}, & G &= mg, & C_y^{\min} &\leq C_y \leq C_y^{\max}, & C_x &= C_{x_0} + k C_y^2, \\ \rho &= \rho_0 e^{-\beta H}, & g &= g_0 \frac{R^2}{(R+H)^2}, & \dot{V} &= -C_x q \frac{S}{m} - g \sin \Theta, \\ \dot{\Theta} &= C_y q \frac{S}{mV} + \left( \frac{V}{R+H} - \frac{g}{V} \right) \cos \Theta, & \dot{H} &= V \sin \Theta, & L &= (R/(R+H)) \cos \Theta. \end{aligned} \quad (2)$$

where  $n_{\Sigma}$  is the full loading factor,  $q$  – the dynamic velocity pressure,  $\rho$  – the atmospheric density,  $V$  – the velocity of the device,  $\Theta$  – the path inclination,  $H$  – the flight altitude,  $L$  – the distance,  $G$  – the weight of the device,  $m$  – the mass,  $g_0$  – the acceleration of gravity on the Earth surface,  $R$  – the radius of Earth,  $C_x$  – the drag force coefficient,  $C_y$  – the lift force coefficient,  $S$  – the typical device area, and  $C$ ,  $C_{x_0}$ ,  $k$ ,  $\rho_0$ ,  $\beta$ ,  $C_y^{\min}$ ,  $C_y^{\max}$  and  $N$  are constants. For the system (1)-(2), the following initial conditions are given:

$$V(0) = V_0, \quad \Theta(0) = \Theta_0, \quad H(0) = H_0, \quad L(0) = 0. \quad (3)$$

and the boundary conditions:

$$L(T) = a, \quad V(T) = V_1, \quad \Theta(T) = \Theta_1, \quad H(T) = H_1, \quad T \text{ is not fixed}, \quad (4)$$

where  $a$  is a parameter. For the calculations we use the maximum principle, the nonlinear programming problem and parallel computations. We also consider the nonregular and degenerated maximum principle. The upper and lower bounds of the parameter  $a$  are defined by maximal and minimal distances.

This work was done with the financial support of the RFBR grant - project code 15-07-08952.

## Visualization of neighborhoods in certain topologies

*Vesna Velickovic*

**Short abstract.** Many interesting topologies arise in the theory of  $FK$  spaces. Here we apply our software package for the visualization to represent neighborhoods in various topologies.

## Spectral properties of the Finslerian multilinear symmetric forms in the physics of Blodgett-Langmuir monolayers

*Vladimir Balan, Halina Krylova and Nina Krylova*

**Abstract.** A recent approach within the spectral theory of symmetric tensors and multivariate homogeneous polynomials addresses the spectral properties of multilinear symmetric forms naturally attached to Finsler structures, e.g. the metric fundamental, and the Cartan tensors [6]. The particular  $m$ -th root structures of locally Minkowski type serve as a preferred illustrative examples to emphasize the complexity of the associated spectral data [1-3]. In the present study, we investigate the case of the symmetric tensors of the natural Finsler-type structures which are related to the interphase boundary phenomena of a monomolecular Langmuir-Blodgett monolayer [4-5, 10]. The  $Z$ -,  $H$ - and  $E$ - and  $N$ -eigenvalues and the corresponding eigenspaces [7-9, 11-13] are determined, their geometric relevance is pointed out, and the best particular Candecomp/Parafac approximants are inferred.

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### References

- [1] V. Balan, *Spectra of multilinear forms associated to notable  $m$ -root relativistic models*, Linear Algebra and Appl. (LAA), 436, 1, 1 (2012), 152-162.
- [2] V. Balan, *Spectral properties and applications of numerical multilinear algebra of  $m$ -th root structures*, Hypercomplex Numbers in Geom. Phys. 2 (10), 5 (2008), 101-107.
- [3] V. Balan, G. Bogoslovsky, S. Kokarev, D. Pavlov, S. Syparov, N. Voicu, *Geometrical models of the locally anisotropic Space-Time*, Hypercomplex Numbers in Geom. Phys., Moscow, 1 (15), 8 (2011), 4-37.
- [4] V. Balan, H. V. Grushevskaya, N. G. Krylova, M. Neagu, A. Oana, *On the Berwald-Lagrange scalar curvature in the structuring process of the LB-monolayer*, Applied Sciences 15 (2013), 30-42.
- [5] V. Balan, H. Grushevskaya and N. Krylova, *Finsler geometry approach to thermodynamics of first order phase transitions in monolayers*, Differential Geometry - Dynamical Systems, 17 (2015), 24-31.
- [6] D. Bao, S.-S. Chern, Z. Shen, *An Introduction to Riemann-Finsler Geometry*, Springer-Verlag, 2000.
- [7] K.C. Chang, K. Pearson, T. Zhang, *On eigenvalue problems of real symmetric tensors*, Journal of Mathematical Analysis and Applications 350 (2009), 416-422.
- [8] A. Cichocki, *Era of big data processing: a new approach via tensor networks and tensor decompositions*, arXiv:1301.6068 (2014).
- [9] L. de Lathauwer, B. de Moor, J. Vandewalle, *A multilinear SVD*, SIAM J. Matrix Anal. Appl. 21 (2000), 1253-1278.
- [10] M. Neagu, N. G. Krylova, H. V. Grushevskaya, *Jet theoretical Yang-Mills energy in the geometric dynamics of 2D-monolayer*, J. Math. Phys. 54 (2013), 1-14.
- [11] L. Qi, *Eigenvalues of a real supersymmetric tensor*, Jour. Symb. Comp. 40 (2005), 1302-1324.
- [12] L. Qi, *Rank and eigenvalues of a supersymmetric tensor, the multivariate homogeneous polynomial and the algebraic hypersurface it defines*, Jour. Symb. Comp. 41 (2006), 1309-1327.
- [13] L. Qi, W. Sun, Y. Wang, *Numerical multilinear algebra and its applications*, Front. Math. China 2 (4) (2007), 501-526.

## A Rosen type bi-metric universe and its physical properties

*Wladimir Georges Boskoff*

**Abstract.** The paper studies a spacetime endowed with two stationary metrics. The first one is a Riemannian one, called the  $R$ -Schwarzschild metric. It satisfies the Einstein vacuum field equations, correctly describes the slow down of clocks in the gravitational field, the orbits of the planets and the perihelion drift. The  $R$ -Schwarzschild metric can be seen as the basic texture of the spacetime. All objects having mass are ruled by this Riemannian metric. The second metric, i.e. the Schwarzschild light-adapted metric, is deduced both taking into account the Rosen bi-metric compatibility condition and by the speed of the light limit axiom preservation. This second metric offers the texture of the "light-like" objects. And the main "normal" surprise is that this metric can be only the classical Schwarzschild metric. So, a Rosen type bi-metric universe exists and all known physical properties are preserved.