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Abstracts [26 August 2018]

Fractional differential equations with some applications

Ricardo Almeida

Short abstract. This talk deals with fractional differential equations, in terms of a fractional derivative with respect to another function. Existence and uniqueness results are proven. A Picard iterative method is exemplified. Then, some applications are presented. We develop some mathematical models to describe the world population growth and the gross domestic product of some countries, for such derivatives.

Hyperbolic second-order partial differential equation-based model for structural interpolation

Tudor Barbu

Abstract. A novel partial differential equation-based structural image interpolation (inpainting) technique is proposed in this work. It is based on a nonlinear hyperbolic PDE model that successfully inpaints the image affected by missing regions by directing the diffusion mostly to the missing part, by using a second-order anisotropic diffusion component and an inpainting mask. The second-order time derivative of the hyperbolic model reduces the diffusion effect from the vicinity of the edges, thus providing much sharper image details. The proposed PDE model is well-posed and it is numerically solved using an iterative finite difference method-based numerical approximation scheme that is stable, consistent to the hyperbolic model and converges to its weak solution. Some interpolation experiments and method comparison, which illustrate the effectiveness of the developed inpainting model, will be also described.

Some metrics on the tangent bundle of a Weyl manifold

Cornelia-Livia Bejan and Ilhan Gul

Short abstract. Starting with a Weyl structure on the base manifold we construct a Weyl structure on the total space of the tangent bundle, whose conformal class of metrics contains the Sasaki metric on *TM*. By using the previously computed curvature tensor field, we state our main result, which characterizes (in terms of the Sasaki metric) both Weyl structures on *M* and on *TM* to be simultaneously Einstein-Weyl.

The Barbosu-Constantin potential: geometrical representation of the manifold from the system origin

Diana Rodica Constantin, Dumitru Pricopi, Valentin Niculescu, Agneta Anthoaneth Mocanu, Erika Verebelyi

Abstract. In the Barbosu-Constantin model, we investigate some geometric aspects related to the manifold from the origin of the field generated by this potential (BC-potential). Starting from the flow motion equations and the first integrals calculated in the origin of the system, we study the manifold in the neighborhood of the origin as a 2D-torus in the 4D-phase space. We perform representations of torus geodesics, equilibrium orbits and some geometrical properties in relation to these features of the torus.

On classification of some non-singular manifolds in the space M(1,3) × R(u) and symmetry reduction of the eikonal equation

Vasyl Fedorchuk, Volodymyr Fedorchuk

Abstract. Symmetry reduction is one of the most powerful tools to investigate partial differential equations with non-trivial symmetry groups. In particular, for this purpose, we can use a classical Lie method. This method, among others, makes it possible to perform symmetry reduction and construction invariant solutions of the above-mentioned equations. To classify the symmetry reductions and invariant solutions of these equations, we use the structural property of the low-dimensional nonconjugate subalgebras of the same rank of the Lie algebras of the symmetry groups of the studied equations.

Geometro-thermodynamic model of cosmological first-order phase transition with axially symmetric metric

Halina Grushevskaya, Nina Krylova

Abstract. A theory of cosmological models with axially symmetric metrics were constructed on base of a contact statistical 5-dimensional manifold, which describes phase transitions proceeding in the interface systems with an electrocapillary mechanism of energy dissipation. These metrics are that of 4-dimensional surfaces in the 5D manifold. We show that the Newman–Unti–Tamburino theory parameter *n* is the gauge parameter of a scalar field which plays the

role of fifth dimension. The geometro-thermodynamic model has been applied to cosmological first-order phase transitions. Nucleation and further evolution of true-vacuum bubbles have been considered.

On the volume of Sasakian manifolds

Oliver Goertsches, Hiraku Nozawa and Dirk Töben

Abstract. This talk is based on the joint work [GNT17], and a work in progress. A Sasakian manifold is characterized as a Riemannian manifold (M,g) with a contact form, whose Riemannian cone ($\mathbb{R}_{>0}\times$M, $dr^2 + r^2g$; $d\eta$) is Kaehler. Typical examples are odd dimensional spheres and the link of certain isolated singularity. The volume of Sasakian manifolds is studied from both mathematical and phyisi-cal point of view. For example, Martelli-Sparks-Yau [MSY] showed that nontrivial variation of the volume of Sasakian manifolds is an obstruction to the existence of compatible Sasaki-Einstein metrics. On the physical aspect, the volume of Sasakian manifolds corresponds to the central charge of a superconformal string theory via the AdS/CFT correspondence.

We talk about some different ways to compute the volume of Sasakian manifolds. After explaining localization formulas due to Martelli-Sparks-Yau [MSY] and us [GNT17], we discuss a remarkable relation of the volume of Sasakian manifolds to the Lefschetz number of the Reeb ow on the Dolbeault cohomology of the cone discovered by Bargman-Herzog [BH02] and Martelli-sparks-Yau [MSY]: For a Sasakian manifold M with resolution W of the singularity at the origin of the cone, let $C(\exp(-tb); W)$ be the Lefschetz number of the Reeb flow $\{\exp(-tb)\}_{t\in\mathbf{R}}$ on $\oplus_k H^{0;k}(W)$. Then we have

$$\mathrm{Vol}(M) = \lim_{t\to 0} t^{(\dim M+1)/2}\, C(\exp(-tb); W).$$

A problem is that $\oplus_k H^{0;k}(W)$ is of infinite dimension and $C(\exp(-tb); W)$ may not be well defined as a function on t. To give a rigorous proof to this formula, we see the Lefschetz number $T_C \to \mathbf{C}$; $g \to C(g; W)$ is convergent on a domain of the complexification T_C of the closure of the Reeb flow, which acts on W.

References

[BH02] A. Bergman, C.P. Herzog, *The volume of some non-spherical horizons and the AdS/CFT correspondence*, J. High Energy Phys. 0201 (2002) 030.

[GNT17] O. Goertsches, H. Nozawa, D. Töben, *Localization of Chern-Simons type invariants of Riemannian foliations*, Israel J. of Math. **222** (2017), 867-920.

[MSY] D. Martelli, J. Sparks and S.-T. Yau, *Sasaki-Einstein manifolds and volume minimisation*, Comm. Math. Phys. **280** (2008), no. 3, 611-673.

Modeling of gene network functioning by dynamical systems

Vladimir Golubyatnikov

Abstract. We study piece wise linear kinetic dynamical systems as models of some gene networks. Discretization of their phase portraits allows to find conditions of existence of their cycles. Consider the following 3-dimensional piece wise linear dynamical systems of the type

$$\frac{dx_1}{dt}=f_1(x_3)-k_1x_1;\quad \frac{dx_2}{dt}=f_2(x_1)-k_2x_2;\quad \frac{dx_3}{dt}=f_3(y_2)-k_3x_3, \qquad (1)$$

where

$$f_j(x_{j-1})=A_j \text{ for } 0\le x_{j-1}<1; f_j(x_{j-1})=0 \text{ for } 1\le x_{j-1}, j=1,2,3$$

as a model of a circular gene network functioning, see [1]. Here all coefficients and variables are positive, the step functions f_j are positive and monotonically decreasing, they describe negative feedbacks.

Theorem. *If $A_1>2k_1$, $A_2>2k_2$, $A_3>2k_3$, then the system (1) has at least one cycle.*

References

[1] M.B. Elowitz, S. Leibler, *A synthetic oscillatory network of transcriptional regulators*, Nature **403** (2000), 335-338.

Non-trivial topological properties of quasi-relativistic model of graphene

Halina Grushevskaya and George Krylov

Short abstract. A quasi-relativistic model of graphene accounted of relativistic exchange interactions has been investigated, regarding the topological properties of its momentum space. It has been explicitly proved the existence of a non-trivial non-abelian Zak phase for the model related with the $\mathbf{Z}_2\times\mathbf{Z}_4$ homotopy group.

Hypersurfaces in E^{n+1} satisfying $\Delta H=\lambda H$

Ram Shankar Gupta

Abstract. We prove that hypersurfaces in E^{n+1} satisfying $\Delta H=\lambda H$, whose second fundamental form has constant norm and with at most four distinct principal curvatures, have constant mean curvature and constant scalar curvature. In particular, every such null 2-type hypersurface in Euclidean space E^{n+1} with at most four distinct principal curvatures has constant mean curvature and constant scalar curvature. Also, we obtain that every such biharmonic hypersurface in E^{n+1} with at most four distinct principal curvatures must be minimal and has constant scalar curvature. Furthermore, the mean curvature H and the constant norm of the second fundamental form β of every such nonminimal hypersurface satisfy $H^2 \leq \beta / n$ and equality holds if and only if M is congruent to $S^n(\lambda/n)$, where $\lambda = \beta$.

Hamilton Ricci flow, Riemann flow and associated geometric dynamics

Iulia Hirică, Constantin Udrişte

Abstract. Hamilton published a groundbreaking paper in 1982, introducing the concept of the Ricci flow. For a given Riemannian manifold (M, g_0), the Ricci flow is a PDE that evolves the metric tensor: $\partial/\partial t\, g(t,x) = -2\, S(g(t,x))$, $g(0,x) = g_0$, where $S(g(t,x))$ denotes the Ricci tensor generated by the metric $g(t,x)$. The idea of Hamilton was to try to evolve the metric in some way, that will make the manifold "rounder and rounder". The notion of Riemann flow is a natural generalization of Ricci flow. Like for any flow, we built the least squares Lagrangian and the associated geometric dynamics via Euler-Lagrange PDE. Then we study some properties of the solutions $g(t,x)$ of the new PDE, corresponding to certain Ricci flow and Riemann flow.

On the historical development of Field Theory in Finsler and Lagrange spaces

Franz-Karl Klepp

Short abstract. In this talk we present a historical development of Field Theory in Finsler spaces. The Finslerian approach is found to be an alternative to the Riemannian general or special relativity. It leads to their extension and it brings now concepts of space-time. This Field Theory by S. Ikeda, G.S. Asanov, R.Miron and other authors are presented. Further, the construction of a Field Theory and its evolution in the Lagrange Geometry are discussed.

Pseudo-Finsler geometro-thermodynamical model of two-dimensional phase transition of the first order with domain-wall distortion under collisions

Nina Krylova, Halina Grushevskaya, Vladimir Balan, George Krylov

Abstract. A model, which describes the dynamics of two-dimensional (2D) first-order phase transition in pseudo-Finsler thermodynamic-configuration space, has been proposed. The Finsler-Lagrange dynamics is studied taking into account the heterogeneity of the nucleation processes, notably observed as a relaxation times distribution of phase nuclei. The Finsler 2D-billiard theory has been proposed to describe a motion of domain walls under an action of interphase collisions.

Anti-MANOVA and applications

Hwiyoung Lee, Vladimir Balan and Victor Pătrăngenaru

Abstract. Object data analysis (ODA) is a collection of statistical methods for analyzing data on manifolds or stratified spaces. In this presentation, the authors consider Fréchet data analysis on compact manifolds. The maximizers of the Fréchet function on such an object space is called Fréchet antimean. Anti-MANOVA is an extension of classical MANOVA in an ODA context. One has to test for the equality of k antimeans on a compact object space with a manifold structure. Here we give a chi-square test statistics for an anti-MANOVA hypothesis testing problem, based on large samples. The low sample size case leads to a nonparametric bootstrap anti-MANOVA test statistics, which are in particular applied to 3D projective shape data, with an application to image analysis.

Measures of noncompactness and their applications

Eberhard Malkowsky

Abstract. We demonstrate how results from the theory of measures of noncompactness and the theory of BK spaces can be applied to establish identities or estimates for the Hausdorff measure of noncompactness of bounded linear operators between the spaces of convergent and absolutely convergent series. These results yield necessary and sufficient conditions for those operators to be compact. Finally, we consider the series-to-series version of a classical result by Cohen and Dunford, which states that regular matrix operators cannot be compact.

Hypoelliptic problems on Carnot groups

Giovanni Molica Bisci

Short abstract. In this talk several existence and multiplicity results for subelliptic equations on Carnot groups. The main tools are group-theoretical arguments on stratified Lie Groups. The talk is mainly based on recent joint works with P. Pucci as a part of a joint research-project started in 2017.

Nilpotent algebras hidden in Monster Tower

Piotr Mormul

Abstract. 20 years ago, H. Sussmann asked if the Goursat distributions were locally nilpotentizable. That is, if they possessed local pairs of vector generators spanning nilpotent Lie algebras over the reals (the existence of such generators is extremely instrumental in robotics - motion planning problems). By now the answer is known: YES. Due pairs of generators have been produced and the nilpotency orders of the resulting algebras (called Kumpera-Ruiz algebras) have been computed. Two natural questions have emerged under way: (i) What Goursat germs are strongly nilpotent? (ii) What are the real dimensions of the Kumpera-Ruiz algebras? The first question is by now basically answered. The second question remains widely open.

Reference

[#] P. Mormul, Kumpera-Ruiz algebras in Goursat flags are optimal in small lengths. J. Math. Sciences 126 (2005), 1614 - 1629.

Spin 1 particle with polarizability in the external Coulomb field, nonrelativistic description

Elena Ovsiyuk, Olga Veko, Yanina Voynova, Viktor Red'kov

Abstract. Quantum-mechanical equation for a spin 1 particle with additional electromagnetic characteristic polarizability is investigated in presence of external Coulomb field. From the 15-dimensional relativistic equation written in Proca tensor form, a corresponding nonrelativistic equation for 3-dimensional wave function is derived. The separation of the variables first is performed in the relativistic equation, with the use of tetrad approach and Wigner D-functions. By applying the discrete operator of spatial reflection, the radial system of 15 linked equations is split into two subsystems: of 5 and 10 equations, respectively. The system of 5 equations reduces to a known and solvable problem, in which the polarizability does not manifest itself in the energy spectrum. From the radial system of 10 equations, in order to simplify the task, we derive the corresponding nonrelativistic system of 2-nd order linked equations for two functions, which further lead to a 4-th order equation; the last has two irregular singular points, both of the rank 3. The states with minimal value of the quantum number of the total angular momentum j=0 are separately considered; for this case the problem in nonrelativistic approximation reduces to a single 2-nd order radial differential equation with two irregular points of the rank 2. For both derived equations of 2-nd and 4-th order, the formal Frobenius solutions are constructed, among these we separate solutions that can be associated with bound states of the vector particle with polarizability in the Coulomb field.

KCC geometric objects for the system describing a spin 1 particle in the Coulomb field

Elena Ovsiyuk, Olga Veko, Yanina Voynova, Vasily Kisel, Vladimir Balan, Viktor Red'kov

Abstract. The emerging system under study consists of 10 radial equations, and is derived from the Duffin–Kemmer–Petiau equation for a spin 1 particle in the external Coulomb field. With the use of the space reflection operator, the whole system is split to independent subsystems, consisting of 4 and 6 equations, respectively. The simplest subsystem of 4 equations is solved in terms of hypergeometric functions, which gives a known energy spectrum. As well, the solutions and the energy spectrum are found for the minimal value of the total angular momentum, j=0. The second subsystem describes two other series of bound states.

By means of the Lorentz generalized condition, in presence of the Coulomb field, we prove that one of the 6 radial functions turns to be equal to zero. This simplifies the explicit form of the system of 6 equations, which contain only 5 unknown functions. By recombining this system, we derive a system of 2-nd order differential equations for three separated radial functions. In particular, one such equation turns out to be rather simple, and turns out to be a confluent Heun equation. Consequently, a series of bound states is constructed in terms of the so called transcendental confluent Heun functions, which provides us with solutions for the second class of bound states, with a corresponding formula for energy levels.

After excluding two non-differential relations from the subsystem of 6 equations, without any additional constraints due to the Lorentz condition, we infer a system of 1-st order differential equations for 4 independent functions f_i, i=1,2,3,4. For this (linear and homogeneous) 1-st order system, we present the KCC geometric framework, and apply them to the second-order natural extension; as well, we describe the geometric objects of the extended system from the perspective of the KCC theory.

New examples and new constructions for skew-symmetric algebroids

Paul Popescu, Marcela Popescu

Short abstract. Our aim is to investigate some properties of a skew symmetric algebroid, i.e. an anchored vector bundle with a corresponding skew symmetric bracket and to study the properties of a simple example, a rare thing in the literature. For the sake of simplicity, we consider the skew symmetric case; however, certain skew symmetric conditions may be removed.

Noether symmetries in presence of nonlinear nonholonomic constraints

Paul Popescu, Marcela Popescu

Short abstract. We construct Noether invariants for Lagrangian nonholonomic dynamics with affine or nonlinear constraints, considered to be adapted to a foliation on the base space. A set of illustrative examples are given, including linear and nonlinear Appell mechanical systems.

Invariant connections on Lie groups

Gabriel Pripoae

Abstract. In this talk, we study the sets of the left invariant and of the bi-invariant connections on Lie groups, endowed with some additional properties: symmetry, flatness, Ricci-flatness, etc. New invariants arise which may be used for the classification of Lie groups and Lie algebras. These invariants are new, are interesting (in our opinion) and, moreover, have a strong geometric flavor. Moreover, we give some new examples in low dimensions for some type of affine connections.

The new Minkowski norm and integral formulae for a manifold with a set of one-forms

Vladimir Rovenski

Abstract. Integral formulas are the power tool for obtaining global results in Analysis and Geometry. We explore the problem: *Find integral formulas for a closed manifold with a set of linearly independent one-forms (or vector fields).* To do this, we introduce new Minkowski norm, determined by Euclidean norm α, linearly independent one-forms β_i, $(1 \le i \le p)$ and a function φ of p variables; this produces a new class of 'computable' Finsler metrics, which reduce to Matsumoto's (α,β)-metric when $p=1$. Then we explore a Riemannian structure on a manifold, naturally arising from our Minkowski norm and one-form $\omega\neq 0$, and find the second fundamental form of the distribution $\operatorname{Ker}\omega$ through invariants of α, ω, β_i and φ. Finally, we apply the above to prove new integral formulae for a closed Riemannian manifold with $\omega\neq 0$ and linearly independent one-forms β_i, $(1 \le i \le p)$, which generalize Reeb's integral formula and its counterpart for the second mean curvature.

Optimal control problems in Differential Geometry

Constantin Udrişte

Short abstract. In this paper we want to lighting what are the optimal control problems in differential geometry and how to solve them. As controls we use either a family of affine connections, or a family of curvature tensor fields, or a family of Ricci tensor fields. The raised problems clearly describe the addressed research.

Multitime optimal control applied to resource economics

Constantin Udrişte, Ionel Ţevy

Abstract. Multitime optimal control theory has been used recently in economics. Here we develop some ideas using the context of optimal control and differential geometry. Specifically, two problems are analyzed: (i) multitime evolution of reproducible resources and (ii) multitime evolution of non-reproducible resources. The technique is to associate with each problem a proper functional and appropriate constraints (geometric PDEs like m-flows, Goursat-Darboux PDEs, parallelism PDEs and specific isoperimetric constraints). When the Hamiltonian is linear affine in the control, we focus on bang-bang and singular optimal controls.

Properties of the fundamental Finsler function

Constantin Udrişte, Ionel Ţevy and Ali Sapeeh Rasheed

Abstract. Consider $\mathbf{R}^n$ as a manifold of dimension n. Classically, a Finsler structure on $\mathbf{R}^n$ is a non-negative function $F : T\,\mathbf{R}^n \equiv \mathbf{R}^n \times \mathbf{R}^n \to R$, $(x,y) \to F(x,y)$ that is smooth and positive away from the zero section of $T\,\mathbf{R}^n$, homogeneous of degree 1 (i.e., $F((x,\lambda y) = |\lambda| F(x,y)$ for all $y \in T_x\,\mathbf{R}^n$ and $\lambda \in \mathbf{R}$), and the partial function $y \to F^2(x,y)$ is Euclidean strictly convex on each tangent space $T_x\,\mathbf{R}^n$ for $x \in \mathbf{R}^n$. It follows that the partial function $y \to F(x,y)$ is Euclidean convex. For the manifold $\mathbf{R}^n$ all tangent spaces are isomorphic one another $T_x\mathbf{R}^n \equiv \mathbf{R}^n$. Generally, let $\varphi : \mathbf{R}^n \times \mathbf{R}^n \to \mathbf{R}$, $(x, y) \to \varphi(x, y)$ and let a set-valued map a: $\mathbf{R}^n \to P(\mathbf{R}^n)$. The function $m(y) = \inf_x \{ \varphi(x,y) \mid x \in a(y)\}$ is called minimum function. Taking into account some properties of minimum functions, here we find some properties of minimum Finsler function $f(y) = \inf_x \{ F(x,y) \mid x \in a(y)\}$, where $y \in T_x\,\mathbf{R}^n$ is identified with $y \in \mathbf{R}^n$.

Roots of economic black holes

Constantin *Udrişte, Massimiliano Ferrara and Ionel Ţevy*

Abstract. Starting from the isomorphism between the concepts and techniques of Thermodynamics and Economics, we describe the "economic black holes" as small parts of a global economic system in which national income is so great that it causes others poor enrichment. Our theory is based on a new flow between two macroeconomic systems (gravity model drew on analogy with Newton's Law of Gravitation) and a Schwarzschild type metric on an economic system.

Some curves and surfaces of given curvature and their graphical representations

Vesna Veličković

Short abstract. We consider certain planar curves with given curvature and the surfaces of revolution generated by them. Furthermore, we establish a relation between the curvature of a curve and the Gaussian curvature of its corresponding surface of revolution and determine some surfaces of revolution with given Gaussian curvature. We use our own software for the graphical representation of the considered curves and surfaces.

Integrability of the geodesic flow on the resolved conifolds over Sasaki-Einstein space $T^{1,1}$

Mihai Vişinescu

Abstract. Methods of Hamiltonian dynamics are applied to study the geodesic flow on the resolved conifolds over the Sasaki-Einstein space $T^{1,1}$. We explicitly construct the constants of motion and prove the complete integrability of geodesics in the five-dimensional Sasaki-Einstein space $T^{1,1}$ and its Calabi-Yau metric cone. The singularity at the apex of the metric cone can be smoothed out in two different ways. Using the small resolution, the geodesic motion on the resolved conifold remains completely integrable. Instead, in the case of the deformation of the conifold, the complete integrability is lost.

Reference

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