Abstracts [vn. 7.0 / 11 october 2019]

Candecomp/Parafac and HO-SVD decompositions in the Finslerian framework

Vladimir Balan

Abstract. The spectral N-way approach has recently provided fruitful extensions for the classic spectral theory of linear operators and, and powerful tools for Big Data and Image Processing. It relevantly enhances the statistical analysis in MRI-imaging, Special Relativity, ecology and HARDI biology. In the present talk, we first provide a brief survey of results from the spectral theory of N-way arrays, and of its applications to fundamental anisotropic geometric structures, and further address two models for which we derive the HO-SVD decomposition and the Parafac/Candecomp approximation - which respectively provide powerful aids for identifying main geometric features, and consistent estimates for the original structures. We discuss as well alternative siblings of the main spectral equations.

Variational and PDE-based image compression models: a survey

Tudor Barbu

Abstract. This paper provides an overview on the partial differential equation-based image compression domain. It describes state of the art PDE-based image compression techniques, such as those using linear homogeneous diffusion-based models and those based on edge-enhancing diffusion (EED), like the EED-based compression schemes using B-tree triangular coding (BTTC) for sparsification and the rectangular subdivision with edge-enhancing diffusion (R-EED) approaches. Also, our own contributions in this field, representing effective compression and decompression solutions using PDE-based edge detection and nonlinear anisotropic diffusion-based inpainting, are presented here and compared with the state of the art methods.

Multidimensional iterative processes with diagonal evolution

Andreea Bejenaru and Monica Pîrvan

Abstract. This paper introduces a new type of multivariate (multidimensional) discrete equations, characterized by diagonal evolution. In particular, the corresponding 2D diagonal system is distinct from classical Roesser's or Fornasini-Marchesini's models. Further, these diagonal discrete equations are used as evolution laws for multivariate discrete optimal control problems. A practical example referring to the maximization of consumption and minimization of waist is provided in order to motivate our approach. Also, the structural stability of the newly defined 2D dynamics is analyzed.

Isometric or harmonic mappings of complete Riemannian manifolds

Tran Quoc Binh

Abstract. We investigate (a) the isometric and (b) the harmonic mappings ϕ of a complete Riemannian manifold M^n whose sectional curvature is bounded from below, into Euclidean space E^{n+m} and in case of (a) also into the unit sphere $S^{n+m-1} \subset E^{n+m}$. In case of (a) we obtain conditions in terms of the Euclidean norm $\|H(\phi(x))\|$, $x \in M^n$ of the mean curvature vector of $\phi(M^n)$ on the radius r of the Euclidean ball $B(r)$ in order that $\phi(M^n)$ cannot be pinched in any such $B(r)(\phi(M^n) \not\subset B(r))$. In case of (b) we show that under a mild condition on the Ricci curvature the positivity of the energy density $e(\phi)$ is necessary in order that $\phi(M^n)$ spreads out to infinity.

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Extrinsic spherical depth in Hilbert manifolds

Seunghee Choi, Victor Patrangenaru and Vladimir Balan

Abstract. The novelty of Object Data Analysis comes from the fact that it contains a myriad of different types of data sets that often do not belong to a Euclidean space. By representing key features extracted from images, Object Data Analysis allows for studying the contents of image data. Often times an object space is nonlinear, therefore it has to be embedded into a Euclidean space. We propose spherical depth function as a nonparametric methodology for estimating the extrinsic mean objects on a manifold embedded in a Hilbert space. As an example, from a colored map of groundwater on the Earth, one might estimate a distribution of groundwater.

Kink-type solitons for the nonlinear model in microtubules

Armando Ciancio, Haci Mehmet Baskonus and Carlo Cattani

Abstract. In this paper, we use the exponential function method to find some complex travelling wave solutions in the nonlinear dynamics model which describes the dimer`s dynamics within microtubules. We obtain some entirely complex and kink-type soliton solutions to this nonlinear model. By choosing some suitable values of parameters, we plot the various dimensional simulations of all the obtained solutions in this study. We observe that our result may be useful in detecting some complex waves behaviors of kink solitons moving along the microtubule.

A thermodynamic model for population growth with relaxation phenomena

Vincenzo Ciancio, Antonella Lupica and Annunziata Palumbo

Abstract. Reaction-diffusion models were used in dynamic fluid, population growth, pulse propagation in nerves and other biological phenomena. Some of these models have been expanded to describe memory effects in diffusion and therefore with the use of hyperbolic equations deriving from the generalization of the Fourier and Fick laws. These generalizations come from the theory of extended irreversible thermodynamics (EIT) which is based on kinetic theory arguments. Recently it has been shown that, using the procedures of the classical irreversible thermodynamics with internal variables (CIT-IV), we can obtain equations for the dissipative flows that generalize the laws of Fourier-Fick and Cattaneo-Vernotte.

In this paper, by using the methodology of CIT-IV, we propose a new model that includes the effect of memory in the diffusion highlighting the presence of relaxation time. The diffusion flow obtained is characterized by the sum of a parabolic and a hyperbolic contribution which allows the formulation of a dynamic system.

The hydrogen like atom: 2D representation of its orbital types

Diana Rodica Constantin, Valentin Ioan Niculescu, Agneta Anthoaneth Mocanu, Flavia Nacu, Erika Varga-Verebelyi

Abstract. In this paper we approach to the topic on the hydrogen like atom with high angular momentum. This type of atom is involved in the cold plasma phenomena. We work in the quantum frame of well known physical model, namely the Schroedinger equation with Coulomb potential. Using the symbolic computation, we evaluate the wave function and the local probability density associated to our problem. We plot a superposition of effective potential with orbital quantum number $l=0..4$. Also, we obtain a 2D representation of the orbital types of the states characterized by the quantum numbers n, $l=n-1$, and $m = 0$ with $n=1..5$.

Multitemporal nonlinear Schrodinger PDE

Cristian Ghiu, Lavinia Laura Petrescu, Constantin Udrişte

Abstract. The multitemporal nonlinear Schrodinger PDE was stated for the first time in our research group as a universal amplitude equation which can be derived via multiple scaling analysis in order to describe slow modulations in multitime and space of the envelope of a spatially and temporarily oscillating wave packet (an equation which governs the dynamics of solitons through meta-materials). Now we exploit some hypothesis in order to find important explicit solutions. Particularly, we discovered a family of ODEs and a family of PDEs whose solutions generate solutions of multitime nonlinear Schrodinger equation.

A review of the geometric nature of Einstein's theory

Graham Hall

Abstract. This talk will give a historical discussion of the genesis of Einstein's general theory of relativity with particular attention given to its geometrical foundations. It will start with a brief discussion of the ideas of gravitation before Einstein (classical Newtonian theory) and then proceed to Einstein's special theory of relativity. From here, Einstein's general relativity can be introduced together with its dependence on differential geometry. Some major aspects of general relativity theory can then be described including the field equations, some exact solutions of these equations and a brief mention of cosmology.

Influence of domain growth on first-order phase transitions of generalized Newman-Unti-Tamburino cosmological model

Halina Grushevskaya, Nina Krylova and George Krylov

Abstract. The first-order electroweak phase transitions at the electroweak scale of standard cosmology exist beyond Standard Models but may solve some cosmological problems, like the generation of the baryon asymmetry of the universe, production of gravitational waves. Modern cosmological models are constructed in the framework of thermodynamic approaches developed within a Van der Waals – Maxwell theory of the first-order phase transitions. In the present work we study interfacial first-order phase transitions with the distribution of relaxation times in a configuration space which describes geometro-thermodynamics of a spacetime with generalized Newman–Unti–Tamburino-like metric. The pseudo-Finsler billiard model has been proposed to describe metric perturbations in the phase transition process due to relativistic domain-wall velocities. It has been found out that the metric perturbations lead to bending geodesics.

Flatness produced by some geometric PDEs

Iulia Hirică, Constantin Udrişte, Gabriel Pripoae, Ionel Ţevy

Abstract. This talk has several goals. The first idea is to consider and study the PDEs of connection-flatness, curvature-flatness, Ricci-flatness, scalar curvature-flatness in a modern and rigorous way. Although the idea is not new, our main Theorems about flatness introduce a new point of view in Differential Geometry. The second idea is to introduce and study the Euler-Lagrange prolongations of PDEs-flatness solutions via associated least squares Lagrangian densities and functionals on Riemannian manifolds. All the geometric PDEs were proved to turn into one of the most intensively developing branches of modern Differential Geometry.

Theory for a spin 3/2 particle in electromagnetic and gravitational fields, spherical symmetry

Alina Ivashkevich, Nina Krylova, Vasily Kisel and Viktor Red'kov

Abstract. The general theory for a spin 3/2 particle in presence of external electromagnetic fields on the background of an arbitrary curved space-time, is elaborated. The 16-component equation in Rarita–Schwinger basis is divided into the main equation and two constraints, algebraic and differential ones. First, this system is solved in spherical coordinates of the Minkowski space-time. There are constructed wave functions on which 4 physical operators are diagonalized: of energy, square and third projection of the total angular momentum, and space reflection; they accordingly correspond to the quantum numbers ε, j, m, P. After separating the variables, the main system of 8 radial equations and additional 2 algebraic and 2 differential constraints, are derived. Their solutions are constructed in the form of linear combinations of Bessel functions, and with the use of the known properties of these functions, the complete system of equations is transformed to a purely algebraic form. Ultimately, the task reduces to one linear constraint on three numerical parameters, $A_1 a_1 + A_2 a_2 + A_3 a_3 = 0$, where the given coefficients A_i are expressed through the quantum numbers ε and j. Any two linearly independent sets $\{a_1, a_2, a_3\}^{1,2}$ determine two different quantum states of the spin 3/2 particle. The study is extended to the case of de Sitter space-time background. The mathematical task turns out to be much more complicated.

Vector particle with anomalous magnetic and electric quadroupole moments in the Coulomb field: separating the variables, nonrelativistic approximation, analysis of Frobenius solutions

Artem Koral'kov, Yanina Voynova, Elena Ovsiyuk and Viktor Red'kov

Abstract. We start with a relativistic Duffin-Kemmer-Petiau equation for spin 1 particle with anomalous magnetic moment in presence of external Coulomb field. By diagonalizing operators of energy, square and third projecting the total angular momentum, we derive a system of 10 radial equations. According to the requirement of diagonalizing the spatial reflection operator, we split the system into subsystems of 4 and 6 equations for states with parities $P = (-1)^{j+1}$ and $P = (-1)^{j}$, respectively. The additional due to anomalous moment interaction terms are present only in the subsystem related to states with parity $P = (-1)^{j}$. In order to simplify the problem, we perform in the subsystem for states with $P = (-1)^{j+1}$ the transition to nonrelativistic approximation, so deriving two linked second order differential equations for two radial functions. States with minimal $j = 0$ are the simplest, being described by a 2nd order equation of double confluent Heun type. Its related bound state Frobenius solutions are studied. By imposing the known transcendency condition, which gives a 4th order algebraic equation with respect to energy, we yield a series of energies having the structure $E = -\text{const}/n^2$. The numerical values for these energies seem to be physically interpretable, though they do not depend on the anomalous magnetic moment, whereas the corresponding radial solutions depend on such a parameter. For states with $j = 1, 2, \ldots$ we derive the 4th order equations for radial functions. There are constructed 4 different solutions for these equations, and the convergence of involved power series with 8- and 9-terms recurrent relations is studied. The

transcendency condition gives the formula for energies $E = -\text{const}/n^2$, which does not depend on the quantum number j and on the parameter of anomalous magnetic moment, and therefore cannot describe the physical spectrum.

A similar analysis is performed for vector particles with different electromagnetic characteristics, electric quadrupole moment, as well. The emerging mathematical problems turn out to be similar, however some physical differences arise. For instance, both subsystems, for states with parities $P = (-1)^{j+1}$ and $P = (-1)^j$, now depend now on the parameter of quadrupole moment. All the constricted solutions are exact, but they are formal because any quantization for energies of bound states are not known.

Spin 1 particle with additional electromagnetic moments in the external Coulomb field, applications of the KCC-theory

Nina Krylova and Vladimir Balan

Abstract. In the present talk we study two quantum-mechanical problem of a spin 1 particle with additional electromagnetic characteristics, anomalous magnetic and electric quadrupole moments, in presence an external Coulomb field. The study of the systems of the relevant differential equations is based on the use of the Kosambi–Cartan–Chern geometrical approach (KCC-theory). In both cases, the first and the second invariants have been calculated. It has been shown that the different branches of the solution converge near the singular points of the systems. This correlates with the behavior of the solutions near three singular points of radial equations for quantum mechanical states. For each case, the set of Lagrangians corresponding to the geometrical problem under consideration has been found. It has been demonstrated that the Lagrangians contains a specific gauge freedom.

*Curvature properties of Sasakian metrics on second order tangent bundle**

Abdullah Magden, Aydin Gezer and Kubra Karaca

Abstract. Let M be an n-dimensional Riemannian manifold with a Riemannian metric g and T^2M be its second-order tangent bundle with Sasaki metric S^g. In this paper, firstly, we calculate Riemannian curvature tensor, scalar curvature and sectional curvature tensor fields of Sasaki metric on T^2M and prove that T^2M is a manifold of constant sectional curvature or of constant scalar curvature, respectively. Then, we search weakly symmetry property of Sasaki metric and we show that it is characterized in terms of flatness of the base metric and Sasaki metric.

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Matrix transformations and fixed point iterations

Eberhard Malkowsky

Abstract. Matrix transformations play an important role in fixed point theory. First, some results are studied concerning the connectedness of sets of limit points of matrix transforms of bounded complex sequences. Of special interest are sufficient conditions for the connectedness of sets of limit points of complex sequences. Furthermore, theorems are considered concerning transforms of bounded complex sequences and their sets of limit points. Finally, we apply these results and the theory of matrix transformations to fixed point iterations defined by the use of infinite matrices, where the focus is on the study of the Mann iteration.

Inequalities for statistical submanifolds

Adela Mihai

Abstract. Statistical manifolds have arisen from the study of statistical distributions. We consider statistical submanifolds of certain statistical manifolds and obtain inequalities involving curvature invariants and Chen-like inequalities. This is a joint work with M.E. Aydin, H. Aytimur, M. Kon, I. Mihai, C. Ozgur and K. Takano.

Parameterizations of Delaunay surfaces from scratch

Ivailo Mladenov and Vladimir Pulov

Abstract. Relying on first principles one can write down the condition that an axially symmetric surface has a constant mean curvature as a first order ordinary differential equation. In the course of one more integration we produce both their internal classification and explicit parameterizations.

Infinitesimal symmetries in the Monster Towers

Piotr Mormul and Fernand Pelletier

Abstract. This work is a continuation of authors' research [2] interrupted in the year 2010. Derived are ([3]) recursive relations describing for the first time all infinitesimal symmetries of special 2-flags (sometimes also misleadingly called 'Goursat 2-flags'; for precise definitions we refer the reader to [1]). When algorithmized to the software level, they will give an answer filling in the gap in knowledge as of 2010: on one side the local finite classification of special 2-flags known in lengths not exceeding four ([2]), on the other side the existence of a continuous numerical modulus of that classification in length seven.

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Riemann-Lagrange geometry for starfish/coral dynamical system

Mircea Neagu

Short abstract. In this paper we develop the Riemann-Lagrange geometry, in the sense of nonlinear connection, d-torsions, d-curvatures and Yang-Mills-like energy, associated with the dynamical system concerning social interaction in colonial organisms. Some possible trophodynamic interpretations are derived.

Classes of concave mappings and related optimization

Octav Olteanu and Janina Mihaela Mihăilă

Abstract. Starting from a strictly increasing, strictly concave continuous real function h defined on $(0,\infty)$, with the properties $h(1)=0$, $h(1+x)<x, \forall x>0$, and being given a bounded sequence $(c_n)_{n\geq 1}$ of real numbers such that $\inf_{n\geq 1} c_n > 0$, one proves that for any convergent series $\sum_{n=1}^{\infty} x_n = M$ of nonnegative numbers, the following inequality holds: $\sum_{n=1}^{\infty} c_n h(1+x_n) \geq \left(\inf_{n\geq 1} c_n\right) h(1+M)$. Modifying the constraint $\sum_{n=1}^{\infty} x_n = M$ into $\sum_{n=1}^{\infty} c_n x_n = M \in (0,\infty)$, also assuming that $u \to u \cdot h(1+1/u)$ is strictly increasing on $(0,\infty)$, one proves that $\sum_{n=1}^{\infty} c_n h(1+x_n) \geq \left(\inf_{n\geq 1} c_n\right) h\left(1+\frac{M}{\inf_{n\geq 1} c_n}\right)$. The idea is to prove such inequalities firstly for finite sums, then passing to the limit. In case of finite sums, minimum principle for concave continuous functions is applied and the optimum point(s) can be determined. Two main examples of such functions h are given: $h_1(t)=ln(t), h_2(t)=t^p-1, 0<p<1, t>0$. In the second part, an operator-valued version of some of the above results is discussed in detail. To prove this main result of the paper, two basic preliminary results on ordered vector spaces of self-adjoint operators are recalled and applied. Some ideas from the first part are adapted to the second part too. A new element is a natural Lipschitz-type condition, which is verified by both classes of operator-valued versions of the above examples. This new element, as well as other basic notions and proofs, plays a central role. A first main nontrivial inequality is $\sum_{n=1}^{\infty} T_n h(I+x_n A) \geq \left(\inf_{n\geq 1} T_n\right) h(I+sA)$ (here $\sum_{n=1}^{\infty} x_n$ is a convergent numerical series of positive terms, $s := \sum_{n=1}^{\infty} x_n$, $\sigma(T_n) \subset [a,b] \subset (0,\infty)$, $n \in \mathbb{N}\backslash\{0\}$, $\sigma(A) \subset (0,\infty)$, h being obtained from a continuous increasing concave real function (denoted by h too), by means of functional calculus, $h(1)=0$.

Gosset´ polytope and del Pezzo surfaces; connections with Physics and Neuroscience

Ovidiu Păsărescu

Abstract. There are several theories in Physics aiming to unify the standard model and gravitation. Two of them are: String Theory and Emergent Quantum Mechanics. We introduce a class of mathematical objects (in nonstandard extension of topoi) which are ¨infons¨ in the first theory and ¨receptons¨ in the second one (pseudo-particles). In the second theory the 4D space-time is ¨pixelated¨ by 4D volumes appearing as projections of the Gosset polytope from the 8-dimensional space, which we build using (-2)-curves on del Pezzo surfaces. Moreover, the consciousness and human thinking is modeled with that pseudo-particles.

Multitemporal KdV solitons

Lavinia Laura Petrescu, Constantin Udrişte

Abstract. The term of multitemporal soliton has recently been coined in our research school to describe a multitemporal pulselike nonlinear wave (multitemporal solitary wave) which emerges from a collision with a similar pulse having unchanged shape and speed vector. To introduce this notion, it was necessary to introduce multitemporal PDEs that extend solitonic single-time PDEs via geometrical ingredients. This paper covers the status for multitemporal soliton research based on multitemporal dual power law nonlinear KdV PDE, paying particular attention to methods whereby an initial value problem for such a PDE can be solved exactly through a succession of calculations. Discussion of the interaction between two bi-temporal solitons show their own physical sense.

On the geometrization of vector fields

Gabriel Teodor Pripoae

Short abstract. We define and study new invariants, for the geometrization of autonomous and non-autonomus vector fields. In particular, we deal with the problem of the existence of affine structures on Lie groups.

Hamiltonian forms for nonholonomic equations with nonlinear constraints

Marcela Popescu and Paul Popescu

Short abstract. Different Hamiltonian forms for nonholonomic equations with nonlinear constraints are presented. Some regularity conditions are considered in the presence of a foliation.

Deforming convex bodies in Minkowski Geometry

Vladimir Rovenski

Abstract. The talk is devoted to deformations of Minkowski norms [3] in $\mathbf{R}^n$, determined by a sequence of p linearly independent 1-forms $_i$ and a positive function ϕ of p variables. This generalizes construction of a Minkowski norm [2]. For $p = 1$, our deformations generalize (α,β)-norm [1], which can be seen as images of a Euclidean norm under our deformations.

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On the cut locus of Randers surfaces

Sorin Vasile Sabău

Abstract. In this talk we shall present some new results on the structure of the cut locus of a surface of revolution endowed with a Randers metric. This Randers metric is introduced as the solution of the Zermelo's navigation problem determined by a Riemannian metric of revolution h and a vector field W that plays the role of the wind. We will consider the cases when the wind is an h-Killing vector field as well as the more general case when the wind can have a more general direction. The relation with the cut locus of the Riemannian metric h will be explained in detail.

On Bi-Slant Submersions

Cem Sayar, Mehmet Akif Akyol and Rajendra Prasad

Abstract. In the current study, we introduce the notion of bi-slant submersion from Kaehler manifolds and give an example to show the existence of this type of submersion. The integrability of the distributions and the geometry of the fibers of such submersions are studied. By considering the canonical structures parallel, certain conclusions are drawn. We give some curvature relations between the base space and the total space.

Entropy of Reissner-Nordstrom 3D black hole in Roegenian economics

Constantin Udrişte, Massimiliano Ferrara, Ionel Ţevy, Dorel Zugrăvescu, Florin Munteanu

Abstract. The subject of this paper is to analyze the Mathematical Principia of Economic 3D Black Holes in Roegenian economics. In details, we study two main problems: (i) mathematical origin of economic 3D black holes, (ii) entropy and internal political stability depending on national income and the total investment, for economic RN 3D black hole. To solve these problems, it was necessary to jump from macroeconomic side to microeconomic side (a substantial approach so different), to complete the thermodynamics-economics dictionary with new entities, and to introduce the flow between two macroeconomic systems. The main contribution is about introducing and studying the Schwarzschild type metric on an economic 4D system, together with Rindler coordinates, Einstein 4D PDEs, and economic RN 3D black holes. In addition, we introduce some economic Ricci type flows or waves, for further research.

Information Geometry in Roegenian Economics

Constantin Udrişte, Ionel Ţevy

Abstract. We characterize the geometry of the statistical Roegenian manifold that arises from the equilibrium distribution of an income of non-interacting identical economic actors. The main results for ideal income (Section 2) are included in three subsections: partition function in distribution, scalar curvature, and geodesics. Although this system displays no phase transition, its analysis provides an enlightening contrast with the results of Section 3, where we shall examine the geometry of the economic Van der Waals income, which does exhibit a "monetary policy as liquidity - income" transition. Section 3 focuses on three subsections: canonical partition function, the economic limit, and information geometry of the economic Van der Waals manifold.

Left invariant Hermitian structures on CH^2 and RH^4

Srdjan Vukmirovic, Marijana Babic and Andrijana Dekic

Abstract. $\mathbf{C}H^2$ and $\mathbf{R}H^4$ are the only 4-dimensional non-compact rank one symmetric spaces. As such, they can be modeled by real solvable Lie groups with left invariant metrics (proven by Heintze). We classify all possible left invariant Hermitian structures on these Lie groups, i.e., Riemannian metrics and Hermitian complex structures. Then we consider corresponding left invariant geometry of these groups.