The XIV-th International Conference of Differential Geometry and Dynamical Systems (DGDS-2020)
27 - 29 August 2020 * ONLINE * [Bucharest, Romania]

List of abstracts

[August 29, 2020]

A characterization for totally real submanifolds in Kaehler product manifolds

Mohd. Aquib

Short abstract. We study totally real submanifolds in Kaehler product manifold with constant scalar curvature using self-adjoint differential operator □. Under this setup, we obtain a characterization result. Moreover, we discuss δ-invariant properties of such submanifolds and get an obstruction result as an application of the inequality derived. We also provide example in the support of the result.

Complete surfaces with zero curvatures in conformally flat spaces

Kelicio Araujo, Armando Corro, Romildo Pina and Marcelo Souza

Abstract. In this talk, we introduce a family of Riemannian manifolds $\mathbf{E}^3{}_F$, which are Euclidean space $\mathbf{R}^3$ endowed with conformally flat metrics. We characterize rotational surfaces with constant Gaussian and extrinsic curvatures in $\mathbf{E}^3{}_F$. We present a particular space that is isometric to $\mathbf{H}^2\mathrm{x}\mathbf{S}^1$, and, using a special parametrization, we construct a family of complete rotational surfaces with zero Gaussian and extrinsic curvatures in $\mathbf{H}^2\mathrm{x}\mathbf{S}^1$. We have built a special space that is a warped product $\mathbf{H}^2\times_f\mathbf{R}$, which is a complete space foliated by complete surfaces of constant Gaussian curvature -1; this shows that the hyperbolic space $\mathbf{H}^2$ is isometrically immersed into the space $\mathbf{H}^2\times_f\mathbf{R}$, and this space is isometric to neither $\mathbf{H}^3$, nor $\mathbf{H}^2\times\mathbf{R}$, showing that in the ambient space $\mathbf{H}^2\times_f\mathbf{R}$ Hilbert theorem does not hold.

Inequalities for statistical submanifolds in cosymplectic statistical manifolds

Mohd. Aslam, Mustafa Kazaz and Mohd. Aquib

Short abstract. In this paper, we obtain Euler's inequality and Chen's inequality for statistical submanifolds in the cosymplectic statistical manifolds with constant curvature and discuss the equality case of the inequalities. We also give some applications of the inequalities obtained.

$(LCS)_n$-manifolds and pseudosymmetric structures

Kanak Kanti Baishya and Manoj Ray Bakshi

Abstract. The talk attempts to study different pseudosymmetric conditions based on the study of equivalence of geometric structures (given by Shaikh and Kundu in 2013) by considering different conditions into various groups or classes in $(LCS)_n$-manifolds. Moreover, we determine the almost quasi-Yamabe soliton in an $(LCS)_n$ -manifold under the above mentioned various groups.

Hyper-generalized weakly symmetric GRW-spacetimes

Manoj Ray Bakshi, Ashoke Das and Kanak Kanti Baishya

Abstract. The authors have proved that the 4-dimensional Lorentzian concircular structure (known as $(CS)_4$-spacetime) coincides with the Generalized Robertson-Walker (GRW) spacetimes. Consequently, to study hyper generalized weakly symmetric $(CS)_4$-spacetimes is equivalent to study the GRW-spacetimes. The present talk attempts to determine the curvature conditions for which the hyper-generalized weakly symmetric GRW-spacetimes are sometimes perfect fluid spacetimes and some other time remain infinitesimally spatially isotropic. Also, we point out the sufficient condition for a compact, orientable hyper-generalized weakly symmetric GRW-spacetime to be conformal to a sphere in the 5-dimensional Euclidean space E_5.

Mathematical models for photon-limited image restoration

Tudor Barbu

Short abstract. A survey of the state of the art mathematical models for photon-limited image restoration is provided in this work. The most important partial differential equation (PDE) - based Poisson denoising techniques are described here. So, quantum noise filtering algorithms using Total Variation (TV)-based models and second- and fourth-order diffusion schemes are presented. Also, our own contributions in this domain, representing effective photon-limited image denoising solutions based on nonlinear parabolic and hyperbolic PDE models, are described and compared to the state of the art approaches.

On pseudo-umbilical spacelike submanifolds in indefinite space form M_{p}^{n+p}(c)

Majid Ali Choudhary

Short abstract. In the present note, first we derive an intrinsic inequality for pseudo-umbilical spacelike submanifolds in an indefinite space form. We use this inequality to show that such submanifold is totally geodesic. In the rest of this paper, using a result of Aiyama, we prove that any pseudo-umbilical spacelike subamnifold is totally umbilical.

Invariant submanifolds of (LCS)n-manifolds admitting certain conditions

Sabina Eyasmin

Short abstract. The object of the talk is to study the invariant submanifolds of (LCS)n-manifolds. We study generalized quasi-conformally semiparallel and 2-semiparallel invariant submanifolds of (LCS)n-manifolds and showed their existence by non-trivial example.

On symmetry reduction and some classes of invariant solutions of the (1+3)-dimensional Monge-Ampère equation

Vasyl Fedorchuk and Volodymyr Fedorchuk

Short abstract. We study the relationship between the structural properties of the low-dimensional (dim $L \leq 3$) nonconjugate subalgebras of the same rank of the Lie algebra of the Poincaré group $P(1,4)$ and the properties of reduced equations for the (1+3)-dimensional Monge-Ampère equation. Some classes of invariant solutions are constructed.

Lie symmetries of the canonical connection: codimension one Abelian nilradical case

Ryad Ghanam and Gerard Thompson

Abstract. This paper is concerned with finding minimal dimension linear representations for six-dimensional real, indecomposable nilpotent Lie algebras. It is known that all such Lie algebras can be represented in gl(6,**R**). After discussing the classification of the 24 such Lie algebras, it is shown that only one algebra can be represented in gl(4,**R**). A Theorem is then presented that shows that 13 of the algebras can be represented in gl(5,**R**). The special case of filiform Lie algebras is considered, of which there are five, and it is shown that each of them can be represented in gl(6,**R**) and not gl(5,**R**). Of the remaining five algebras, four of them can be represented minimally in gl(5,**R**).

Linear discrete multitime multiple recurrence

Cristian Ghiu, Raluca Tuligă and Constantin Udrişte

Short abstract. The multitime multiple recurrences are common in analysis of algorithms, computational biology, information theory, queueing theory, filters theory, statistical physics etc. The theoretical part about them is little or not known. Therefore, the aim of our paper is to formulate and solve problems concerning nonautonomous multitime multiple recurrence equations. Among other things, we discuss in detail the cases of linear recurrences with constant coefficients, highlighting in particular the theorems of existence and uniqueness of solutions.

Best approximation of flatness on Riemannian manifolds

Iulia Elena Hirică, Constantin Udrişte, Gabriel Teodor Pripoae and Ionel Ţevy

Abstract. The behavior of many different systems in nature and science are governed by PDEs. The most important geometric PDEs are those producing flatness (e.g. connection-flatness, curvature-flatness, Ricci-flatness, scalar curvature-flatness) and those producing constant curvature (−1,0,1). The purpose of this work is to introduce and study the Euler-Lagrange prolongations of flatness PDEs solutions (best approximation of flatness) via associated least squares Lagrangian densities and functionals on Riemannian manifolds. Also, we develop techniques to find intrinsic properties of PDEs. Conformal changes of metrics are considered and explicit examples are discussed.

Yamabe solitons on Riemannian manifolds

Shyamal Kumar Hui

Abstract. As a tool for constructing metrics of constant scalar curvature in a given conformal class of Riemannian metrics on a Riemannian manifold (M^n, g), n ≥3, R. S. Hamilton was introduced the notion of Yamabe flow, which is an evolution equation for metrics on M as:

(1) $$\frac{\partial g}{\partial t} = -rg$$

where r is the scalar curvature corresponds to g. The Yamabe flow and the Ricci flow are equivalent in case of 2-dimensional Riemannian manifold. However, in case of dimension n >2, the Yamabe and Ricci flows do not agree as the first one preserves the conformal class of the metric but the Ricci flow does not in general.

A Yamabe soliton is a special solution of the Yamabe flow that moves by one parameter family of diffeomorphisms generated by a fixed (time-independent) vector field V on M and homothetic. A Yamabe soliton on a Riemannian manifold (M,g) is a triplet (g,V,σ) such that

$$\frac{1}{2}\pounds_V g = (r - \sigma)\mathrm{g},$$

where $\pounds_V$ denotes the Lie derivative in the direction of the vector field V and σ is a constant. The Yamabe soliton is said to be shrinking, steady and expanding according as σ < 0, = 0 and > 0 respectively. If σ is a smooth function on M then the metric satisfying (1) is called almost Yamabe soliton. It may be noted that Yamabe solitons coincide with the Ricci solitons in dimension n=2 and for n>2, the Ricci solitons and Yamabe solitons have different behaviours. In this talk, we will discuss the Yamabe solitons on M, whose potential vector field is torse forming and the evolution of some geometric quantities on a compact Riemannian manifold M whose metric is Yamabe soliton. We will also introduce (k,m)-quasi Yamabe gradient solitons.

Solving spinor Maxwell equations in cylindrical parabolic coordinates, and spinor space structure

Alina Ivashkevich and Viktor Red'kov

Abstract. Maxwell equations in any Riemannian space-time can be presented in spinor form on the base of tetrad method, when the Maxwell field is described by local 2nd rank symmetrical spinor. This general covariant equation is specified in cylindrical parabolic coordinates and corresponding diagonal tetrad. After separating the variables, we derive the system of four 1-st order differential equations in partial derivatives for three functions, depending on two parabolic coordinates. The mathematical task reduces to one 2nd order equation in partial derivative for a main function, which determining all remaining functions. Solutions are constructed in terms of the confluent hypergeometric functions. We study the properties of four types of constructed solutions - they must be continuous and single-valued - in the context of vector and a spinor space model. It I shown that in space with vector structure only two variants provide us with correct solutions; in space with spinor all four variants are appropriate. It is shown that diagonalization of the helicity operator for 2-rank symmetric spinor it follows the system of equations coincided with that following from the Maxwell equations, when identifying the eigenvalue with the frequency of electromagnetic solutions. Two eigenvalues, zero and negative, of the helicity operator must be ignored.

Applications of Finsler geometry to the theory of fields

Francisc Klepp

Abstract. In this talk we refer, in historical sequence, to some applications of Finsler Geometry in the Theory of Fields. We cover the wide range between the mathematical model of the electron microscope - presented by Roman Ingarden in 1957 - and the unification of electrodynamics and gravitation by means of Finslerian structures. In the geometries based on Lagrangians, like Finsler or Lagrange geometry, the so called deflection tensor field is strongly involved. Its significance for Finsler geometry was pointed by Makoto Matsumoto when he formulated the well-known axioms determining the Cartan connections of a Finsler space. In his studies on Lagrange geometry Radu Miron has used the deflection tensor field for the construction of a geometrical model in which gravitation and electromagnetism are unified, while having a metrical connection with non-vanishing deflection tensor field.

Spin 1 particle with electric quadrupole moment in the external Coulomb field

Nina Krylova, Yanina Voynova, Elena Ovsiyuk and Vladimir Balan

Abstract. The problem of vector particle with quadrupole moment in the presence of the Coulomb field is studied. We derive systems of 4 and 6 equations, for states with parities $P=(-1)^{j+1}$ and $P=(-1)^{j}$. The system of 4 equations reduces to a 2-nd order equation, its local Frobenius solutions are constructed. Condition of transcendency of solutions gives some quantization rule for energy levels which seem to be physically appropriate. The system of 6 equations turns out to be very complicated. In order to simplify the problem, we perform nonrelativistic approximation. So, deriving two linked 2-nd order equations for two functions, whence it follows a 4-th order equation, its four different Frobenius type solutions are constructed. Transcendency condition gives some formula for energies, which does not depend on the parameter of quadrupole moment and therefore cannot describe physical spectrum correctly. To study the mathematical task under consideration, we additionally apply the geometrical method based on the use Kosambi-Cartan-Chen invariants. The first and the second invariants are calculated. The explicit Lagrangian related to this problem is determined, the Lagrangian has the arbitrariness up to certain terms, which may be considered as specific gauge freedom.

Non-existence of non-trivial warped product lightlike submanifolds of semi-Riemannian product manifolds

Sangeet Kumar and Megha Pruthi

Short abstract. In present paper, we set out to examine warped product lightlike submanifolds of semi-Riemannian product manifolds . Significantly, we analyze warped product GCR-lightlike submanifolds of the type $N_\perp \times_\lambda N_T$ and $N_T \times_\lambda N_\perp$and obtain their non-existence in a semi-Riemannian product manifold $\bar{N}$ where N_T and $N_\perp$, respectively, are holomorphic submanifolds and totally real submanifolds of $\bar{N}$.

Geometric characteristics of screen slant lightlike submanifolds

Tejinder Kumar, Megha Pruthi, Sangeet Kumar and Pankaj Kumar

Short abstract. The aim of present paper is to analyze geometric characteristics of screen slant lightlike submanifolds of indefinite nearly Kaehler manifolds. We establish the existence theorem for screen slant lightlike submanifolds in indefinite nearly Kaehler manifolds. We also derive conditions for the integrability of distributions for such submanifolds. Consequently, we find several characterization results for totally umbilical screen slant lightlike submanifolds in indefinite nearly Kaehler manifolds. Subsequently, minimal screen slant lightlike submanifolds of indefinite nearly Kaehler manifolds are also investigated.

Topological and differential invariants of G-structures with singularities

Mikhail Malakhaltsev and Fabian Antonio Arias Amaya

Short abstract. A principal G-bundle with singularities is a principal bundle π: $\bar{P} \to M$ with structure group $\bar{G}$, which reduces to a subgroup $G \subset \bar{G}$ on the set $M \setminus \Sigma$, where M is an n-dimensional compact manifold and $\Sigma \subset M$, is a k-dimensional submanifold. For example, a vector field on an n-dimensional Riemannian manifold M defines a reduction of the orthonormal frame bundle of M to the subgroup $\mathbf{O}(n-1) \subset \mathbf{O}(n)$ on the set $M \setminus \Sigma$, where Σ is the set of zeros of this vector field. The aim of this talk is to explain how to construct topological and differential invariants of G-structures with singularities and give some examples of computation of these invariants.

Regularity versus Compactness

Eberhard Malkowsky

Abstract. A well-known result by Cohen and Dunford (1937, [1]) characterizes the class of all regular compact linear operators. Consequently, a regular matrix transformation cannot be compact. This means that if c denotes the set of all complex sequences of complex numbers then an infinite matrix that maps c into c and preserves the limits cannot be compact. We gave new a proof of this result which applies the theory of BK spaces from functional analysis and summability and uses the Hausdorff measure of noncompactness. Furthermore, we present similar recent results involving the spaces of sequences that are strongly summable by the Cesàro method of order 1 with index p, and strongly convergent, respectively.

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Matrix Lie groups as 3-dimensional almost paracontact almost paracomplex Riemannian manifolds

Mancho Manev and Veselina Tavkova

Short abstract. Lie groups considered as three-dimensional almost paracontact almost paracomplex Riemannian manifolds are investigated. In each basic class of the classification used for the manifolds under consideration, a correspondence is established between the Lie algebra and the explicit matrix representation of its Lie group.

On hypersurfaces in a hyperbolic space

Yoshio Matsuyama

Abstract. By $H^{n+1}(c)$ we denote an $n+1$ − dimensional hyperbolic space with constant curvature c. Let M be an n−dimensional, connected, locally hyperbolic space $H^n(c)$ with constant curvature c and complete Riemannian manifold and let $f : M \to H^{n+1}(c)$ be an isometric immersion. Then $f(M)$ is of the form $\gamma \times H^{n-1}(c)$, where $H^{n-1}(c) \not\subset \mathbf{R}^n$ is hyperbolic space of $H^{n+1}(c) \not\subset \mathbf{R}^{n+2}$ and γ is a curve $-\infty < s < \infty \to \gamma(s)$ in $\mathbf{R}^2$ perpendicular to $\mathbf{R}^n$.

Complexity of the magnetic braids can be measured by their orthogonal parquets

Faik Mayah, Ali Sapeeh Rasheed and Nisreen Alokbi

Abstract. We study a braided model for the magnetic field and we discuss the behaviour of the complexity of the field lines according to the structure of the (u,v)−field “orthogonal parquet”. We have presented several criteria to measure and understand the complexity of field lines of magnetic fields, in particular, magnetic braids. The first attempt is relating the complexity of the magnetic field with the nature of the skeleton of the orthogonal parquet, especially its critical points. We define an index for the singular points and demonstrate with the help of a complex function that the indices of singular points have integer or half-integer value. We also introduce a simple method to visualise and count the singular points and their indices.

From the Hahn-Banach theorem to the Markov moment problem and isotonicity of convex operators over a convex cone

Mihaela Janina Mihăilă and Octav Olteanu

Short abstract. Recent results on Markov moment problem and Mazur-Orlicz theorem are discussed. Some of the linear solutions are Markov operators.On the other hand, polynomial approximation on unbounded subsets is applied to deduce the existence and uniqueness of the solution of some full Markov moment problems. A truncated moment problem is also under attention.In the end, characterization of isotonicity of a convex operator defined over a convex cone in terms of subdifferentials is studied. Numerous illustrating examples are presented. Determining or evaluating the norms of the involved linear operators via continuity properties of the dominating convex operators is also an aim of this work.

Bref résumé. Cette oevre présent les résultats récents sur le problème du moment de Markov et le théorème de Mazur-Orlicz. Certaines des solutions linéaires sont des opérateurs de Markov. D'autre part, une approximation polynomiale sur des sous-ensembles sans bornes est appliquée pour déduire l'existence et l'unicité de la solution de certains problèmes complets de moment de Markov. Une attention particulière est également portée à un problème tronqué de moments. À la fin, on étude la caractérisation de l'isotonicité d'un opérateur convexe défini sur un cône convexe en termes de sous-différentiels. L'article présente également de nombreux exemples illustratifs. Un autre objectif de cet article est de déterminer ou d'évaluer les normes des opérateurs linéaires impliqués à travers les propriétés de continuité des opérateurs convexes dominants.

Curvature inheritance and torse-forming curvature inheritence symmetry in Finsler spaces

Chayan Kumar Mishra

Abstract. Katzin et al. [G. H. Katzin, J. Levine, and W. R. Davis, J. Math. Phys. 10, (1969), 617] introduced curvature collineations (CC), defined by a vector, satisfying $L_v R^i_{jkh} = 0$, where R^i_{jkh} is the Riemann curvature tensor of a Riemannian space V_n and L_v denotes the Lie derivative. They proved that a CC is related to a special conformal motion which implies the existence of a covariant constant vector field. Unfortunately, recent study indicates that the existence of a covariant constant vector restricts V_n to a very rare special case with limited physical use. In particular, for a fluid space time with special conformal motion, either stiff or unphysical equations of state are singled out. Moreover, perfect fluid space times do not admit special conformal motions. This information was not available in 1969, when CC symmetry was introduced. CC is generalized to another symmetry called "curvature inheritance" (CI) satisfying $L_v R^i_{jkh} = 2\alpha R^i_{jkh}$, where α is a scalar function. They prove that a proper CI (i.e., $\alpha \neq 0$) has direct interplay with the physically significant proper conformal motions. S. P. Singh extended the concept of curvature inheritance in Finsler space and projective curvature inheritance in Finsler Spaces. The aim of this paper is to discuss the curvature inheriting symmetry in Finsler spaces and the torse-forming curvature inheritance symmetry in Finsler spaces.

Special multi-flags at the crossroads of algebraic geometry and differential geometry

Piotr Mormul

Abstract. There exist different approaches to the singularities of special multi-flags, also called "generalized Goursat flags". They live in Monster Towers, in Algebraic Geometry called Semple Towers. Colley, Kennedy et al, in Michigan Math. Journal 166 (2017), have concluded a series of earlier algebro-geometric constructions, by several authors, of fine stratifications of the stages of Semple Towers, eventually producing the so-called "RV-classes of singularities". Earlier the author, in SIGMA 5 (2009), constructed, in the stages of Monster Towers the so-called `singularity classes', using purely differential and Lie-algebraic tools. It had been generally believed that the former classes (much more numerous) were a refinement of the latter ones. This belief now turns out to be false. The two approaches appear, to a sizeable degree, to mutually complement each other.

A distinguished geometry perspective on multi-time affine quadratic Lagrangians

Mircea Neagu

Short abstract. For a space endowed with a general quadratic multi-time Lagrangian and an associated non-linear connection, the paper constructs the main Riemann-Lagrange distinguished geometric objects (linear connection, torsion and curvature).

Dirac particle in the Coulomb field on the background of hyperbolic Lobachevsky model

Elena Ovsiuyk, Artem Koral'kov, Aleksandr Chichurin and Viktor Red'kov

Abstract. The known systems of radial equations describing relativistic hydrogen atom on the base of Dirac equation in Lobachevsky hyperbolic space, is investigated. The relevant 2-nd order differential equation has six regular singular points, and its solutions of Frobenius type are constructed. To produce the quantization rule for energy values, we use the known condition for separating transcendental Frobenius solutions. This provides us with an energy spectrum which is physically interpretable and similar to the spectrum arising from the scalar Klein-Fock-Gordon equation, in this geometrical model. The spectrum coincides with that previously found when studying the same radial equation within semi-classical method. The convergence of the involved series is analytically and numerically proved. The squared integrability of solutions is numerically verified. The visualization of results is included. All the results are extended to the similar problem on the background of spherical Riemann space.

An Introduction to Topological Object Data Analysis

Vic Patrangenaru, Shen Chen and Adam Dixon

Abstract. Topological Object Data Analysis (TODA), is a new area of Statistical Inference for Object Data based on the topological structure of random objects, or of their distributions on an object space. This analysis uses both differential topology and algebraic topology methods. In this talk our focus is on the interface of algebraic topology methods and applied Statistics, based on persistence diagrams. Here one investigates the statistical properties of a type of topological data analysis known as persistent homology; given a random sample

$X_1, X_2, \ldots, X_n$ from an unknown probability measure Q on an object space M, one may "estimate" the homology of the support of Q, by taking the union of balls centered at X_i, i=1,...,n, where the radius of the balls is to be determined. Persistent homology allows one to summarize the homologies in various dimensions, over a principal idea domain, for all radii, in what is known as a persistence diagram. This method was used by Fasy et al (2014) [2], to derive confidence sets for various quantities related to persistent homology. A useful derivative of persistence diagrams is the landscapes approach by Bubenik (2015) [1], and its use in analysis of big and complex data (see, e.g., Kovacev-Nikolic et al. (2016) [3], Petrov et al. (2017) [5], Patrangenaru et al. (2019) [4], Shen and Patrangenaru (2020) [6]). Applications of TODA are also considered here.

References

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On the geometric structures of Hom-Lie groups and Hom-Lie algebras

Esmaeil Peyghan

Abstract. Considering the notion of Hom-Lie groups we describe geometric structures, pseudo-Riemannian metric, Hom-Levi-Civita connection and left invariant almost (para)-contact metric on them. Also, we give the notions of left-invariant normal almost (para)-contact and left-invariant (para)-Sasakian structures on Hom-Lie groups ((para)-Sasakian Hom-Lie algebras). The left-invariant curvature tensor properties of left-invariant para-Sasakian Hom-Lie groups are investigated. Furthermore, we study left-invariant (almost) Kenmotsu structures on Hom-Lie groups (or, almost Kenmotsu Hom-Lie algebras). It is proved that if the Ricci tensor of Kenmotsu Hom-Lie algebras is η-parallel, then the scalar curvature is constant. We describe η-Einstein Kenmotsu Hom-Lie algebras. Then we show that an involutive Kenmotsu Hom-Lie algebra is not Einstein if it carries Ricci-semisymmetric property.

The work done by an isotropic vector force field along an isotropic curve

Dimitar Razpopov and Georgi Dzhelepov

Short abstract. We consider a 4-dimensional Riemannian manifold M with a metric g and an endomorphism Q, whose fourth power is the identity, and Q acts as an isometry on g. An associated metric g' on (M, g, Q) is determined by both structures g and Q. The metric g' is necessarily indefinite and it induces isotropic vectors in the tangent space at an arbitrary point of M. The physical forces are represented by vector fields. We study forces whose vectors are in the tangent space of (M, g, Q). We calculate the corresponding physical work done by arbitrary forces along arbitrary curves with respect to g'. Moreover, we suppose that the vector force fields are isotropic and that they act along isotropic curves. We calculate the physical work done by such forces.

Spinor Maxwell equations in Riemannian space-time and the geometrical modeling of constitutive relations in electrodynamics

Viktor Red'kov, Elena Ovsiyuk, Vasily Kisel and Alina Ivashkevich

Abstract. It is known that the vacuum Maxwell equations, considered on the background of a pseudo-Riemannin space-time may be interpreted as Maxwell equations in Minkowski space, but specified in some effective medium, whose constitutive relations are determined by the metric of the curved space-time. In this context, we will consider space-time models with event horizon. All these models are endowed with a metric of a certain structure, while we restrict ourselves to the spherically symmetric case and consider de Sitter, anti de Sitter, and Schwarzschild models. We also study the hyperbolic Lobachevsky and the spherical Riemann models, parameterized by the coordinates with spherical and cylindric symmetry. We prove that in all the examined cases, the effective tensors of electric permittivity (ε_{ij}) and magnetic permeability (μ_{ij}) both obey the same condition, $\varepsilon_{ij}(x)\mu_{jk}(x)=\delta_{ik}$. The simplicity of expressions for these tensors $\varepsilon_{ij}(x)$ and $\mu_{jk}(x)$ is misleading, since for each of the curved space-time models we need to solve Maxwell equations separately. We shall explicitly construct these solutions, using the Maxwell equations in spinor form.

On the curvature of singular distributions

Vladimir Rovenski and Paul Popescu

Short abstract. We consider a Riemannian manifold endowed with a singular (or regular) distribution, determined as an image of the tangent bundle under smooth endomorphism. Following construction of an almost Lie algebroid, we define the modified covariant and exterior derivatives and their adjoint operators on tensors. We introduce the Weitzenböck type curvature operator on tensors, prove the certain decomposition formula, and derive the Bochner-Weitzenböck type formula. These allow us to obtain vanishing theorems about the null space of the Hodge type Laplacian.

On the geometrization of vector fields

Gabriel-Teodor Pripoae and Cristina-Liliana Pripoae

Abstract. We look for geometric objects associated, in canonical ways, to vector fields on differentiable manifolds, such as: affine connections, (semi)-Riemannian metrics, etc. This approach is motivated by the need of geometrization of autonomous ODEs systems (and, to some extent, of the non-autonomous ones). In this context, we define and study new (differential and affine differential) invariants. Examples are given to support our claims: (i) for the Newtonian vector field; for some left invariant vector fields on Lie groups: (iii) for the Polya vector fields associated to holomorphic functions in the complex plane.

Curvature properties of some spacetimes

Absos Ali Shaikh

Short abstract. In the literature of differential geometry, there are various themes of research. The recent trends of research of modern mathematics are abstraction, generalization, existence, characterization and application. A spacetime is a connected 4-dimensional Lorentzian manifold. Curvature plays a crucial role to determine the shape of a space and curvature of a space is determined by its metric tensor. A manifold is said to be of constant curvature if it is of constant sectional curvature and its Riemannian-Christoffel curvature R is in the form R = kG, where k is a constant and G is the Gaussian curvature (where G = 1/2 (g ∧ g) with Kulkarni-Nomizu product '∧'). During the last eight decades the notion of manifold of constant curvature has been weakened by many researchers throughout the globe in different directions such as locally symmetric spaces by Cartan (1926, Bull. Soc. Math. Fr.), semisymmetric spaces by Cartan (1946, Lecons sur la geometrie des espaces de Riemann), recurrent manifolds by Ruse (1949, Proc. London Math. Soc.), weakly symmetric spaces by Selberg (1956, Indian J. of Math.), generalized recurrent manifolds by Dubey (1979, Indian J. Pure Appl. Math.), pseudosymmetric spaces by Adam´ow and Deszcz (1983, Demonstr. Math.), pseudosymmetric spaces by Chaki (1987, An. Stiint. ale Univ. AL. I. Cuza din Iasi N. Ser. Sect. Ia), weakly symmetric spaces by Tam´assy and Binh (1989, Coll. Math. Soc. J. Bolyai), hyper generalized recurrent manifolds by Shaikh and Patra (2010, Arch. Math. (Brno)), quasi-generalized recurrent manifolds by Shaikh and Roy (2010, Math. Pannonica), weakly generalized recurrent manifolds by Shaikh and Roy (2011, Ann. Univ. Sci. Budapest. Etvs Sect. Math.) etc. The above processes of generalization are concerned with the first order or second order covariant derivatives of various curvature tensors and such generalized structures are called 1 "Curvature restricted geometric structures". We note that pseudo-symmetric manifold by Chaki is different from pseudosymmetric manifold by Deszcz and weakly symmetric manifold by Selberg is different from that by Tamassy and Binh. As a generalization of semi-symmetric spaces, the notion of pseudosymmetric spaces arose during the study of geodesic mappings by Adam´ow and Deszcz. This is called pseudosymmetric spaces by Deszcz or Deszcz-symmetric spaces. The geometric interpretation of such space is also given by Haesen and Verstraelen (2004, J. Math. Physics; 2007, Manuscripta Math.; 2009, SIGMA). Hence during the last three decades Deszcz-symmetric space is an important topic of research in differential geometry. The main objective of this lecture is to investigate various curvature restricted geometric structures of various spacetimes such as Som-Raychaudhuri spacetime, pure radiation spacetime, Robinson-Trautman spacetime, Nariai Spacetime, Melvin Spacetime etc.

Non-existence of certain type of convex functions on a Riemannian manifold with a pole

Absos Ali Shaikh, Chandan Kumar Mondal and Izhar Ahmad

Short abstract. The study of convex function is an integral part to investigate various geometric and topological properties of a Riemannian manifold. The notion of convex function in a Riemannian manifold is an early concept but it has been taken its modern shape by the study of C. Udrişte. This paper is devoted to the study of non-existence of certain type of convex functions on a Riemannian manifold with a pole. To this end, we have developed the notion of odd and even functions on a Riemannian manifold with a pole and proved the non-existence of non-trivial and non-negative differentiable odd convex function whose gradient is complete. Finally, we have deduced an inequality related with convex function.

On generalized almost statistical convergence in Banach spaces

Absos Ali Shaikh and Biswa Ranjan Datta

Short abstract. In the present work, we introduce the generalized almost statistical (GAS) convergence of real sequences, which is a generalization of almost convergence and statistical convergence of bounded real sequences. Also, we have obtained some results on GAS convergence.

Rectifying and osculating curves on a smooth surface

Absos Ali Shaikh and Pinaki Ranjan Ghosh

Short abstract. The present work studies rectifying and osculating curves on a smooth surface. We find a sufficient condition for which an osculating curve on a smooth surface remains invariant under isometry of surfaces and also we prove that the component of the position vector of an osculating curve $\alpha(s)$ on a smooth surface along any tangent vector to the surface at $\alpha(s)$ is invariant under such an isometry.

Geometry on the surface of revolution with first approximate slope metric

Gauree Shanker and Seema Jangir

Short abstract. The aim of the present work is to study globally defined slope metrics surfaces of revolution. Also, an introduction to the first approximate slope metric and its geodesic behaviour on the surface of revolution is investigated.

Semi-invariant lightlike submanifolds of metallic semi-invariant manifolds

Gauree Shanker, Ramandeep Kaur Aulakh and Ankit Yadav

Short abstract. In this work, we find some conditions for integrability of distributions. Furthermore, we investigate totally geodesic distributions.

Geometry of totally contact umbilical lightlike submanifolds of indefinite Sasakian manifolds

Gauree Shanker, Ankit Yadav and Ramandeep Kaur Aulakh

Short abstract. We prove that every totally contact umbilical proper screen-slant lightlike submanifold of an indefinite Sasakian manifold is totally contact geodesic. Further, we prove the non-existence of totally contact umbilical proper screen-slant lightlike submanifolds of an indefinite Sasakian form.

r-almost Newton-Yamabe solitons on Legendrian submanifolds of Sasakian space forms

Mohd Danish Siddiqi and Sudhkar Kumar Chaubey

Short abstract. In this research paper, we develop the geometrical bearing on Legendrian submanifolds of Sasakian space forms in terms of *r*-almost Newton-Yamabe Soliton with the potential function ψ: $M_n \to \mathbf{R}$. Also, we examine certain conditions for *L*-minimal and totally geodesic Legendrian submanifolds of Sasakian space form admitting the *r*-almost Newton-Yamabe Soliton. Finally, we illustrate some examples based on this study.

Almost Ricci-Bourguignon solitons and geometrical structure in a relativistic perfect fluid spacetime

Mohammad Danish Siddiqi and Aliya Naaz Siddiqui

Short abstract. The present study is based on the geometrical bearing of relativistic perfect fluid spacetime in terms of almost Ricci-Bourguignon solitons and almost η-Ricci-Bourguignon solitons in perfect fluid spacetime with torse-forming vector field ξ. A condition for the almost Ricci-Bourguignon solitons to be steady, expanding or shrinking is also given. In particular, when the potential vector field ξ of the soliton is of gradient type, we derive from the almost η-Ricci-Bourguignon solitons equation a Poisson-Laplacian equation. Finally, we provide an example of 4-dimensional relativistic spacetime admitting the almost Ricci-Bourguignon solitons and almost η-Ricci-Bourguignon solitons.

Rogue wave solutions in integrable systems

Sudhir Singh

Short abstract. A new (2+1)-dimensional integrable Boussinesq model is under investigation. The infinite solutions of the considered model are discussed by using Hirota's bilinear form and a recursive polynomial test function, more specifically, the first, second and third-order rogue wave solutions are explored, and its evolution dynamics is discussed.

On CW-translations of a homogeneous Finsler space with (α,β)-metrics

Sarita Rani Singla and Gauree Shanker

Short abstract. In the present work, we consider a homogeneous Finsler space with two (α,β)-metrics. Taking a Killing vector field X on one of these spaces, we find necessary and sufficient conditions for X to be Killing on the other space. Further, by taking a Killing vector field X of constant length on one of these spaces, we find the condition under which the Killing vector field X has constant length w.r.t. other space also. By Killing vector fields of constant length, we will find CW translations on these spaces.

Z-symmetric manifold with conharmonic curvature tensor

Ayse Yavuz Tasci and Fusun Özen Zengin

Short abstract. The object of the present paper is to study Z-symmetric manifolds with conharmonic curvature tensor. We prove some theorems about such manifolds by using the properties of the Z-tensor.

Geometry of the gamma manifold

Beatrice-Elena Toader

Short abstract. The main objective of this paper is to study the gamma manifold from a geometrical point of view. It is described the space of gamma distributions endowed with a Fisher metric and exponential connections, it's natural coordinate systems, potential functions, the affine immersion in $\mathbf{R}^3$ and the canonical and Kullback-Leibler divergences.

Geometric Dynamics and Wind Theory

Constantin Udrişte, Ionel Ţevy, Simona Dinu and Lavinia Laura Petrescu

Abstract. In this paper we present some applications selected from science and engineering that refer to the movement of particles in fields, geometric dynamics and wind theory. These are usually formulated with the help of dynamical systems that translate physical laws of evolution. As models of winds we introduced and described the double pendulum wind and the *ABC* wind.

Nonholonomic antennas

Constantin Udrişte, Ionel Ţevy and Florin Munteanu

Abstract. Generically, antennas can be designed to transmit and receive radio waves in all horizontal directions equally (omnidirectional antennas), or preferentially in a particular direction (directional, or high-gain, or "beam" antennas). The aim of our paper is to underline the possibility of building optimal antenna as collection of wires (nonholonomic wires), selecting the emitter from the algebraic surfaces gallery. This selection allows the transition from the mathematical description to the concrete realization of the receiver. The pairs "optimal emitter - optimal receiver" are of the following type: (surface, surface), (surface, set of wires), (set of wires, surface), (set of wires, set of wires). Of course, the holonomic mirror means the mirror in the form of a surface and the nonholonomic mirror means a special collection of wires starting from a point. A nonholonomic curve can fill a domain. In this paper we select the emitter from the algebraic surfaces gallery and research the optimal receiver in each case. For emitters like Ţiţeica, Ufo, Wigwom and Subway, we obtain optimal receivers as wires collection type (nonholonomic surfaces), respectively. The Diabolo emitter (surface) corresponds to an optimal receiver of surface type.

Line graphics vs. polygon mesh vs. ray tracing

Vesna Velickovic

Short abstract. We give a comparative comparison of three approaches to the graphical representation of mathematical objects in 3D space. We compare the Line graphics that we develop ourselves with the well-known Polygon mesh and Ray tracing.

Mathematical foundations for a Finsler extension of Einstein gravity theory

Nicoleta Voicu, Christian Pfeifer and Manuel Hohmann

Abstract. In modern field theory, fields are treated as sections of a certain fibered manifold (Y,π,X), called the configuration manifold, Lagrangians are treated as differential forms on some jet bundle J^rY and variations of the action are expressed as Lie derivatives of the Lagrangian with respect to certain vector fields on J^rY. The present talk proposes a framework which allows us to apply the above machinery in constructing well-defined Finsler gravity actions. The main point is the construction of a configuration bundle whose sections are Finsler functions. It turns out that this configuration bundle sits over the positive (or oriented) projective tangent bundle of the spacetime manifold. A concrete vacuum action is then obtained starting from an argument by Pirani and using the variational completion algorithm. Variation of this action with respect to the Finsler function turns out to lead to a scalar equation already proposed in 2011 by Pfeifer and Wohlfarth - this time, having filled the missing pieces in mathematical rigor; in the particular case of Lorentzian metrics, this scalar equation becomes equivalent to the set of vacuum Einstein equations. Coupling to matter is also briefly discussed.

References

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