Pointwise semi-slant submersion

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Abstract. We introduce pointwise semi-slant submersions from almost Hermitian manifolds onto Riemannian manifolds. The geometry of foliation and the integrability of distributions are researched.

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1 Introduction

Riemannian submersions represent an important field of study in geometry and mathematical physics [11, 14]. Riemannian submersions between Riemannian manifolds were defined by O'Neill[15] and Gray [8]. Such submersions are considered between manifolds with differentiable structure were intensively studied (e.g., see [7]). The generalization of such mappings for the case of Hermitian manifolds was studied by Watson [22]. If F is a C^∞ submersion between Riemannian manifolds, there are some variations according to the conditions on F; for instance Riemannian submersions [8, 15], slant submersions [17], anti-invariant Riemannian submersions [18], semi-slant submersions [16, 2], semi-invariant submersions [19], hemi-slant submersions [21, 3], conformal submersions [1], Lagrangian submersion [20], pointwise semi-slant submersions [13], etc.

Şahin [17] studied slant submersions from almost Hermitian manifolds to Riemannian manifolds. If F is a Riemannian submersion from an almost Hermitian manifold to a Riemannian manifolds, then it is called a slant submersion if the angle $\theta(X)$ between JX and the space $kerF_*$ is constant for any nonzero vector $X \in \Gamma(kerF_*)$. The angle θ is called the slant angle of the slant submersion. Further, Park and Prasad defined semi-slant submersions [16]. After that, a lot of studies on these submersions were developed [9, 10, 12].

On the other hand, as a generalization of slant submanifolds, Etayo [6] defined pointwise slant submanifolds. Later, Lee and Şahin [13] introduced pointwise slant submersions. Aykurt Sepet and Ergüt [4] studied pointwise slant submersions from cosymplectic manifolds.

In this paper we introduced pointwise semi-slant submersions from almost Hermitian manifolds to Riemannian manifolds. In this respect, we study the geometry of foliations and the integrability of distributions.

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2 Preliminaries

In this section we recall basic facts on almost Hermitian manifolds and Riemannian submersions.

An almost complex manifold is a manifold M with an almost complex structure, given by a tensor field J of type $(1,1)$ such that $J^2 = -I$. An almost Hermitian manifold (M, J, g) is an almost complex manifold (M, g) with a J-invariant Riemannian metric g. The J-invariance of g means that

$$g(X,Y) = g(JX, JY) \tag{2.1}$$

for any $X, Y \in (TM)$. An almost Hermitian manifold is called Kählerian manifold if

$$(\nabla_X J)\,Y = 0$$

for $X, Y \in \Gamma\,(TM)$, where ∇ is the operator of Levi-Civita covariant differentiation.

Let (M, g) and (N, g') be Riemannian manifolds, with $dim(M) = m$ and $dim(N) = n$ and $m > n$. A Riemannian submersion $F : M \longrightarrow N$ is a map from M onto N satisfying the following two axioms

- F has maximal rank
- The differential F_* preserves the lengths of the horizontal vectors.

For each $q \in N$, $F^{-1}(q)$ is an $m - n$ dimensional submanifold of M, so-called fiber. If a vector field on M is always tangent (or orthogonal) to fibers then it is called vertical (or horizontal)[15]. A vector field X on M is said to be basic if it is horizontal and F-related to a vector field X_* on N, i.e., $F_* X_p = X_{*F(p)}$, for all $p \in M$. We denote the projection morphisms on the distributions $(\ker F_*)$ and $(\ker F_*)^\perp$ by $\mathcal{V}$ and $\mathcal{H}$, respectively.

A Riemannian submersion $F : M \longrightarrow N$ determines two $(1,2)$ tensor fields $\mathcal{T}$ and $\mathcal{A}$ on M. These tensor fields are called the fundamental tensor fields or the invariants of F. For arbitrary vector fields E and F on M, these tensor fields can be given by the formulas

$$\mathcal{T}(E,F) = \mathcal{T}_E F = \mathcal{H}\nabla_{\mathcal{V}E}\mathcal{V}F + \mathcal{V}\nabla_{\mathcal{V}E}\mathcal{H}F \tag{2.2}$$

$$\mathcal{A}(E,F) = \mathcal{A}_E F = \mathcal{V}\nabla_{\mathcal{H}E}\mathcal{H}F + \mathcal{H}\nabla_{\mathcal{H}E}\mathcal{V}F \tag{2.3}$$

where ∇ is the Levi-Civita connection of (M, g). On the other hand for $X, Y \in \Gamma\left((\ker F_*)^\perp\right)$ and $U, W \in \Gamma\,(\ker F_*)$, these tensor fields satisfy the following equations

$$\mathcal{T}_U W = \mathcal{T}_W U \tag{2.4}$$

$$\mathcal{A}_X Y = -\mathcal{A}_Y X = \frac{1}{2}\mathcal{V}[X,Y]. \tag{2.5}$$

It is easy to see that for any $E \in \Gamma\,(TM)$, $\mathcal{T}$ is vertical, $\mathcal{T}_E = \mathcal{T}_{\mathcal{V}E}$ and $\mathcal{A}$ is horizontal, $\mathcal{A}_E = \mathcal{A}_{\mathcal{H}E}$.

Note that a Riemannian submersion $F : M \longrightarrow N$ has totally geodesic fibers if and only if $\mathcal{T}$ identically vanishes.

We recall the following Lemma from O'Neill [15], which is used throughout this paper.

Lemma 2.1. *Let $F : M \longrightarrow N$ be a Riemannian submersion between Riemannian manifolds. If X and Y are basic vector fields of M, then*

1. $g(X,Y) = g'(X_*, Y_*) \circ F$,
2. *the horizontal part $[X,Y]^{\mathcal{H}}$ of $[X,Y]$ is a basic vector field and corresponds to $[X_*, Y_*]$ i.e. $F_*\left([X,Y]^{\mathcal{H}}\right) = [X_*, Y_*]$,*
3. *$[V,X]$ is vertical for any vector field V of $(\ker F_*)$,*
4. *$\left(\nabla^M_X Y\right)^{\mathcal{H}}$ is the basic vector field corresponding to $\nabla^N_{X_*} Y_*$,*

where ∇^M and ∇^N are the Levi-Civita connection on M and N, respectively.

On the other hand, from (2.2) and (2.3) we have

$$\nabla_V W = \mathcal{T}_V W + \bar{\nabla}_V W \tag{2.6}$$
$$\nabla_V X = \mathcal{H}\nabla_V X + \mathcal{T}_V X \tag{2.7}$$
$$\nabla_X V = \mathcal{A}_X V + \mathcal{V}\nabla_X V \tag{2.8}$$
$$\nabla_X Y = \mathcal{H}\nabla_X Y + \mathcal{A}_X Y \tag{2.9}$$

for $X, Y \in \Gamma\left((\ker F_*)^{\perp}\right)$ and $V, W \in \Gamma(\ker F_*)$, where $\bar{\nabla}_V W = \mathcal{V}\nabla_V W$. Moreover, if X is basic then $\mathcal{H}\nabla_V X = \mathcal{A}_X V$.

Let (M,g) and (N,g') be Riemannian manifolds and let $\psi : M \longrightarrow N$ be a smooth mapping between them. The second fundamental form of ψ is given by

$$\nabla\psi_*(X,Y) = \nabla^{\psi}_X \psi_*(Y) - \psi_*\left(\nabla^M_X Y\right) \tag{2.10}$$

for $X, Y \in \Gamma(TM)$, where ∇^{ψ} is the pullback connection. Recall that ψ is said to be harmonic if $\nabla\psi_* = 0$ and ψ is called a totally geodesic map if $(\nabla\psi_*)(X,Y) = 0$ [5]. Finally, we provide the definition of semi-slant submersions.

Definition 2.1. A Riemannian submersion $F : M \longrightarrow N$ is called a semi-slant submersion if there is a distribution $D_1 \subset \ker F_*$ such that

$$\ker F_* = D_1 \oplus D_2, \qquad J(D_1) = D_1 \tag{2.11}$$

and the angle $\theta = \theta(X)$ between JX and the space $(D_2)_p$ is constant for nonzero $X \in (D_2)_p$ and $p \in M$, where D_2 is the orthogonal complement of D_1 in $\ker F_*$. The angle θ is called semi-slant angle.

3 Pointwise semi-slant submersions

Definition 3.1. A Riemannian submersion $F : M \longrightarrow N$ is called a pointwise semi-slant submersion if there is a distribution $D_1 \subset \ker F_*$ such that

$$\ker F_* = D_1 \oplus D_2, \qquad J(D_1) = D_1 \tag{3.1}$$

and for $p \in M$ and $X \in (D_2)_p$, the angle $\theta = \theta(X)$ between JX and the space $(D_2)_p$ is independent of the choice of the nonzero vector X, where D_2 is the orthogonal complement of D_1 in $\ker F_*$. The angle θ is called pointwise semi-slant function of the slant submersion.

Let F be a pointwise semi-slant submersion from an almost Hermitian manifold (M, g, J) onto a Riemannian manifold (N, g'). Then for $U \in \Gamma(\ker F_*)$, we get

$$U = PU + QU \tag{3.2}$$

where $PU \in \Gamma(D_2)$ and $QU \in \Gamma(D_1)$.
For $U \in \Gamma(\ker F_*)$, we obtain

$$JU = \phi U + \omega U \tag{3.3}$$

where $\phi U \in \Gamma(\ker F_*)$ and $\omega U \in \Gamma\left((\ker F_*)^\perp\right)$.
For $X \in \Gamma\left((\ker F_*)^\perp\right)$, we have

$$JX = BX + CX \tag{3.4}$$

where $BX \in \Gamma(\ker F_*)$ and $CX \in \Gamma\left((\ker F_*)^\perp\right)$.

Example 3.2. Let J be a complex structure on $\mathbb{R}^8$ as follows

$$J(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) = (x_2, -x_1, x_4, -x_3, x_6, -x_5, x_8, -x_7).$$

Define a map $F : \mathbb{R}^8 \to \mathbb{R}^4$ by

$$F(x_1, ..., x_8) = \left(\frac{x_1 + x_3}{\sqrt{2}}, \sin\alpha x_6 + \cos\alpha x_7, \frac{x_2 + x_4}{\sqrt{2}}, x_8\right)$$

where $\alpha : \mathbb{R}^8 \to \mathbb{R}$ is any real valued function. Then

$$\ker \pi_* = span\left\{V_1 = \frac{1}{\sqrt{2}}\left(\frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_3}\right), V_2 = \cos\alpha\frac{\partial}{\partial x_6} - \sin\alpha\frac{\partial}{\partial x_7},\right.$$
$$\left. V_3 = \frac{1}{\sqrt{2}}\left(\frac{\partial}{\partial x_2} - \frac{\partial}{\partial x_4}\right), V_4 = \frac{\partial}{\partial x_5}\right\}.$$

Thus the map F is a pointwise semi-slant submersion such that

$$D_1 = \{V_1, V_3\} \quad \text{and} \quad D_2 = \{V_2, V_4\}$$

with the slant function $\theta = \alpha$.

Proposition 3.1. *Let (M, g, J) be an almost Hermitian manifold and (N, g') a Riemannian manifold. $F : (M, g, J) \to (N, g')$ is a pointwise semi-slant submersion if and only if*

$$\phi^2 W = -\left(\cos^2\theta\right) W$$

for $W \in \Gamma(D_2)$, where θ is the slant function.

Proof. The proof of this theorem is similar to Proposition 2.9 in [16] □

Theorem 3.2. *Let F be a pointwise semi-slant submersion from a Kählerian manifold (M, g, J) onto a Riemannian manifold (N, g'). Then the distribution D_1 is integrable if and only if*

$$B\left(\mathcal{T}_U JV - \mathcal{T}_V JU\right) = \phi\left(\hat{\nabla}_V JU - \hat{\nabla}_U JV\right)$$

for $U, V \in \Gamma(D_1)$.

Proof. For $U, V \in \Gamma(D_1)$ and $W \in \Gamma(D_2)$, since $[U, V] \in \Gamma(\ker F_*)$, by using the equation (2.1) we get

$$g([U, V], W) = g(J[U, V], JW) = g(\nabla_U JV - \nabla_V JU, JW).$$

From (2.6) we have

$$g([U, V], W) = g\left(\mathcal{T}_U JV + \hat{\nabla}_U JV\, JW\right) - g\left(\mathcal{T}_V JU + \hat{\nabla}_V JU\, JW\right)$$

By using the equations (3.3) and (3.4), we infer

$$g([U, V], W) = g\left(\mathcal{B}\left(\mathcal{T}_U JV - \mathcal{T}_V JU\right), W\right) + g\left(\phi\left(\hat{\nabla}_U JV - \hat{\nabla}_V JU\right), W\right)$$

Then, we obtain the following result: □

Theorem 3.3. *Let F be a pointwise semi-slant submersion from a Kählerian manifold (M, g, J) onto a Riemannian manifold (N, g'). Then the distribution D_2 is integrable if and only if*

$$\mathcal{T}_Z \omega\phi W - \mathcal{T}_W \omega\phi Z = \phi\left(\mathcal{T}_W \omega Z - \mathcal{T}_Z \omega W\right)$$

for $W, Z \in \Gamma(D_2)$ and $U \in \Gamma(D_1)$.

Proof. For $W, Z \in \Gamma(D_2)$ and for $U \in \Gamma(D_1)$, we have

$$\begin{aligned} g([W, Z], U) =& g(\nabla_W JZ - \nabla_Z JW, JU) \\ =& -g(J\nabla_W \phi Z, U) + g(\nabla_W \omega Z, JU) + g(J\nabla_Z \phi W, U) \\ &- g(\nabla_Z \omega W, JU) \end{aligned}$$

Using Theorem 3.1, we get

$$\begin{aligned} g([W, Z], U) =& \cos^2\theta g([W, Z], U) - g(\nabla_W \omega\phi Z, U) + g(\nabla_W \omega Z, JU) \\ &+ g(\nabla_Z \omega\phi W, U) - g(\nabla_Z \omega W, JU) \end{aligned}$$

Then we arrive at

$$\sin^2\theta g([W, Z], U) = g\left(\mathcal{T}_Z \omega\phi W - \mathcal{T}_W \omega\phi Z, U\right) + g\left(\mathcal{T}_W \omega Z - \mathcal{T}_Z \omega W, JU\right)$$

Thus, the claim is proved. □

Theorem 3.4. *Let F be a pointwise semi-slant submersion from a Kählerian manifold (M, g, J) onto a Riemannian manifold (N, g'). Then the distribution $(\ker F_*)$ defines a totally geodesic foliation if and only if*

$$\begin{aligned}\sin^2\theta g([U,X],V) =& \cos^2\theta g(\mathcal{V}\nabla_X QU, V) + 2\cos\theta\sin\theta X(\theta) g(PU, PV)\\ &+ g(\mathcal{A}_X\omega\phi PU, V) - g(\mathcal{A}_X\omega PU, \phi V) - g(\mathcal{A}_X JQU, \omega V)\\ &- g(\mathcal{H}\nabla_X\omega PU, \omega V) - g(\mathcal{V}\nabla_X JQU, \phi V)\end{aligned}$$

for $U, V \in \Gamma(\ker F_*)$ and $X \in \Gamma\left((\ker F_*)^\perp\right)$.

Proof. For $U, V \in \Gamma(\ker F_*)$ and $X \in \Gamma\left((\ker F_*)^\perp\right)$ by using the equations (2.6) and (3.2) we have

$$g(\mathcal{T}_U V, X) = -g([U,X],V) - g(\nabla_X JPU, JV) - g(\nabla_X JQU, JV)$$

From (3.3) we obtain

$$\begin{aligned}g(\mathcal{T}_U V, X) =& -g([U,X],V) + g\left(\nabla_X\phi^2 PU, V\right) + g(\nabla_X\omega\phi PU, V)\\ &- g(\nabla_X\omega PU, JV) - g(\nabla_X JQU, JV)\end{aligned}$$

Using Theorem 3.1 we infer

$$\begin{aligned}g(\mathcal{T}_U V, X) =& -g([U,X],V) - \cos^2\theta g(\nabla_X PU, V) + 2\cos\theta\sin\theta X(\theta) g(PU, V)\\ &+ g(\nabla_X\omega\phi PU, V) - g(\nabla_X\omega PU, JV) - g(\nabla_X JQU, JV)\end{aligned}$$

Then we arrive at

$$\begin{aligned}g(\mathcal{T}_U V, X) =& -\sin^2\theta g([U,X],V) - \cos^2\theta g(\nabla_U X, V) + \cos^2\theta g(\nabla_X QU, V)\\ &+ 2\cos\theta\sin\theta X(\theta) g(PU, V) + g(\nabla_X\omega\phi PU, V)\\ &- g(\nabla_X JQU, JV) - g(\nabla_X\omega PU, JV)\end{aligned}$$

From (2.8) and (2.9) and since $\mathcal{T}$ is skew-symmetric, we get

$$\begin{aligned}\sin^2\theta g(\mathcal{T}_U V, X) =& -\sin^2\theta g([U,X],V) + \cos^2\theta g(\mathcal{V}\nabla_X QU, V)\\ &+ 2\cos\theta\sin\theta X(\theta) g(PU, V) + g(\mathcal{A}_X\omega\phi PU, V)\\ &- g(\mathcal{A}_X\omega PU, JV) - g(\mathcal{H}\nabla_X\omega PU, JV) - g(\mathcal{A}_X JQU, JV)\\ &- g(\mathcal{V}\nabla_X JQU, JV)\end{aligned}$$

Considering $(\ker F_*)$ as being totally geodesic, we obtain the formula given in the theorem. □

Theorem 3.5. *Let F be a pointwise semi-slant submersion from a Kählerian manifold (M, g, J) onto a Riemannian manifold (N, g'). Then the distribution D_1 defines a totally geodesic foliation if and only if*

$$g(\mathcal{T}_U JV, CX) = -g\left(\hat{\nabla}_U JV, BX\right)$$

and

$$g(\mathcal{T}_U V, \omega\phi W) = \cos^2\theta g\left(\hat{\nabla}_U V, W\right) + g(\mathcal{T}_U JV, \omega W)$$

for $U, V \in \Gamma(D_1)$, $W \in \Gamma(D_2)$ and $X \in \Gamma\left((\ker F_*)^\perp\right)$.

Proof. For $U, V \in \Gamma(D_1)$ and $X \in \Gamma\left((\ker F_*)^\perp\right)$, we obtain

$$g(\nabla_U V, X) = g(\nabla_U JV, JX) = g(\mathcal{T}_U JV, CX) + g\left(\hat{\nabla}_U JV, BX\right)$$

Then we have

$$g(\mathcal{T}_U JV, CX) = -g\left(\hat{\nabla}_U JV, BX\right).$$

On the other hand, for $U, V \in \Gamma(D_1)$ and $W \in \Gamma(D_2)$, we get

$$g(\nabla_U V, W) = g(\nabla_U JV, JW) = \cos^2\theta g\left(\hat{\nabla}_U V, W\right) - g(\mathcal{T}_U V, \omega\phi W) + g(\mathcal{T}_U JV, \omega W)$$

Thus we arrive at

$$g(\mathcal{T}_U V, \omega\phi W) = \cos^2\theta g\left(\hat{\nabla}_U V, W\right) + g(\mathcal{T}_U JV, \omega W).$$

□

Theorem 3.6. *Let F be a pointwise semi-slant submersion from a Kählerian manifold (M, g, J) onto a Riemannian manifold (N, g'). Then the distribution D_2 defines a totally geodesic foliation if and only if*

$$g(\mathcal{A}_X\omega\phi W, Z) = \sin^2\theta g([W, X], Z) - 2\cos\theta\sin\theta X(\theta) g(W, Z) + g(\mathcal{A}_X\omega W, \phi Z) + g(\mathcal{H}\nabla_X\omega W, \omega Z)$$

and

$$g(\mathcal{T}_W\omega\phi Z, U) = g(\mathcal{T}_W\omega Z, JU)$$

for $W, Z \in \Gamma(D_2)$, $U \in \Gamma(D_1)$ and $X \in \Gamma\left((\ker \pi_*)^\perp\right)$.

Proof. For $W, Z \in \Gamma(D_2)$ and $X \in \Gamma\left((\ker \pi_*)^\perp\right)$, we have

$$g(\mathcal{T}_W Z, X) = -g([W, X], Z) - g(\nabla_X W, Z) = -g([W, X], Z) - g(\nabla_X \phi W, JZ) - g(\nabla_X \omega W, JZ).$$

By using Theorem 3.1, we obtain

$$\sin^2\theta g(\mathcal{T}_W Z, X) = -\sin^2\theta g([W, X], Z) + 2\cos\theta\sin\theta X(\theta) g(W, Z) + g(\mathcal{A}_X\omega\phi W, Z) - g(\mathcal{A}_X\omega W, \phi Z) - g(\mathcal{H}\nabla_X\omega W, \omega Z)$$

Then we infer

$$g(\mathcal{A}_X\omega\phi W, Z) = \sin^2\theta g([W, X], Z) - 2\cos\theta\sin\theta X(\theta) g(W, Z) + g(\mathcal{A}_X\omega W, \phi Z) + g(\mathcal{H}\nabla_X\omega W, \omega Z).$$

Also, for $W, Z \in \Gamma(D_2)$ and $U \in \Gamma(D_1)$, we have

$$g(\nabla_W Z, U) = g(\nabla_W JZ, JU)$$
$$= -g\left(\nabla_W \phi^2 Z, U\right) - g(\nabla_W \omega\phi Z, U) + g(\nabla_W \omega Z, JU)$$

Considering Theorem 3.1 and the equation (2.7), we arrive at

$$\sin^2\theta g(\nabla_W Z, U) = -g(\mathcal{T}_W \omega\phi Z, U) + g(\mathcal{T}_W \omega Z, JU).$$

This completes the proof of the theorem. □

Theorem 3.7. *Let F be a pointwise semi-slant submersion from a Kählerian manifold (M, g, J) onto a Riemannian manifold (N, g). If the tensor w is parallel, then we have*

$$\mathcal{T}_{\phi W}\phi W = -\cos^2\theta \mathcal{T}_W W$$

where $W \in \Gamma(D_2)$.

Proof. For $W, Z \in \Gamma(D_2)$, we get

$$\mathcal{T}_W \phi Z = C\mathcal{T}_W Z$$

Similarly, by interchanging the roles of W and Z, we infer

$$\mathcal{T}_Z \phi W = C\mathcal{T}_Z W$$

Then, we can write

$$\mathcal{T}_W \phi Z = \mathcal{T}_Z \phi W$$

By substituting Z by ϕW, we obtain the result. □

Theorem 3.8. *Let F be a pointwise semi-slant submersion from a Kählerian manifold (M, g, J) onto a Riemannian manifold (N, g'). Then F is a totally geodesic map if and only if*

$$-\cos^2\theta \mathcal{T}_U PV + \mathcal{H}\nabla_U \omega\phi PV + C\mathcal{H}\nabla_U \omega PV + \omega\mathcal{T}_U \omega PV$$
$$+ C\mathcal{T}_U JQV + \omega\hat{\nabla}_U JQV = 0,$$

for $U, V \in \Gamma(\ker F_*)$ and $X \in \Gamma\left(\ker F_*^{\perp}\right)$.

Proof. For $X_1, X_2 \in \Gamma\left(\ker F_*^{\perp}\right)$, because of F is a Riemannian submersion and using the equation (2.10), we get

$$(\nabla F_*)(X_1, X_2) = 0.$$

For $U, V \in \Gamma(\ker F_*)$, by considering the (2.10) and (3.3), we have

$$(\nabla F_*)(U, V) = F_*(J\nabla_U JV)$$
$$= F_*(J\nabla_U J(PV + QV))$$
$$= F_*\left(-\cos^2\theta\nabla_U PV + 2\cos\theta\sin\theta U[\theta]PV\right.$$
$$\left.+ \nabla_U \omega\phi PV + J\nabla_U \omega PV + J\nabla_U JQV\right)$$

Using (2.6) and (2.7), we obtain the following equation

$$\begin{aligned}(\nabla F_*)(U,V) = & F_*\Big(-\cos^2\theta\Big(\mathcal{T}_U PV + \hat{\nabla}_U PV\Big) + 2\cos\theta\sin\theta U[\theta]PV \\ & + \mathcal{T}_U\omega\phi PV + \mathcal{H}\nabla_U\omega\phi PV + \phi\mathcal{T}_U\omega PV + \omega\mathcal{T}_U\omega PV \\ & + B\mathcal{H}\nabla_U\omega PV + C\mathcal{H}\nabla_U\omega PV + J\mathcal{T}_U JQV + J\hat{\nabla}_U JQV\Big).\end{aligned}$$

Thus the proof of theorem is completed. □

4 Conclusions

In the present paper we study pointwise semi-slant submersions from almost Hermitian manifolds. Characterizations for such submersions are obtained.

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